CBSE Class 9 Mathemaics

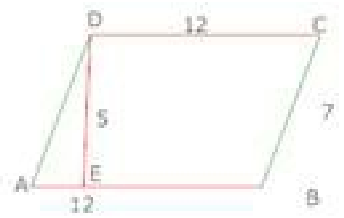
Important Questions

Chapter 9

Areas of Parallelograms and Triangles

1 Marks Quetions

1. Find the area of parallelogram in the adjoining figure.



(a) 1759 foot

(b) 48 square

(c) 84 square foot

(d) 60 square foot

Ans. (d) 60 square foot

2. Find the measure of angle a

(a) $45°$

(b) $60°$

(c) $40°$

d) $65°$

Ans. (b) $60°$

3. A triangle has an area of 45 square foot. Base of the triangle is 9 foot. What is corresponding height of triangle

(a) 90 foot

(b) 5 foot

(c) 10 foot

(d) 40 square foot

Ans. (c) 10 foot.

4. What is area of parallelogram whose base=8 and corresponding altitude is 5

(a) 40

(b) 45

(c) 13

(d) 3

Ans. (a) 40

5. Parallelograms on the same base and between the same parallels have equal

(i) corresponding angle

(ii) area

(iii) congruent area

(iv) same parallel

Ans. (ii) area

6. Any side of a parallelogram is called

(i) Altitude

(ii) base

(iii) corres. Altitude

(iv) area

Ans. (ii) base

7. A diagonal of a parallelogram divides into ______ triangles of equal area

(i) 1

(ii) 2

(iii) 3

(iv) none of these

Ans. (ii) 2

8. Find the area of parallelogram, if Base = 3 and altitude is 4

(i) 7

(ii) 1

(iii) 12

(iv) none of these

Ans. (iii) 12

9. Find the area of 11||gm, if base = 8 cm and altitude = 10 cm,

(a) 80 sq.cm

(b) 80 cm

(c) 30 sq.cm

(d) 50 sq.cm

Ans. (a) 80 sq.cm

10. If Base = 9 and corresponding altitude = 4. Find area of ||gram

(a) 4

(b) 40

(c) 36

(d) none of these

Ans. (c) 36

11. If a triangle and a Parallelogram are on the same base and between the same parallel, the area of the triangle is equal to ______ that of ||gram.

(a) $\frac{1}{2}$

(b) $\frac{1}{3}$

(c) $\frac{1}{4}$

(d) none of these

Ans. (a) $\frac{1}{2}$

12. A parallelogram has an area of 36 square an and base of the parallelogram is 9 cm.

what is the corresponding altitude of parallelogram?

(a) 6 cm.

(b) 5 cm.

(c) 4 cm.

(d) 3 cm.

Ans. (c) 4 cm.

13. A median of a triangle divides if into _______triangles of equal areas.

(a) 1

(b) same triangle

(c) 2

(d) none

Ans. (c) 2

14. The area of a rhombus is equal to _______ of the product of its two diagonals.

(a) $\frac{1}{2}$

(b) $\frac{1}{3}$

(c) $\frac{1}{4}$

(d) none

Ans. (a) $\frac{1}{2}$

15. Area of a triangle is half the product of any of its sides and the

(a) Corresponding altitude

(b) altitude

(c) median

(d) base

Ans. (a) Corresponding altitude

16. Given below are the measurements of a parallelogram. Find the missing measurement. Area = 90 square cm, Base = 5 cm, Height =?

(a) 18

(b) 450

(c) 85

(d) 15 cm

Ans. (a) 18

17. How many square feet are in a square yard

(a) 6

(b) 9

(c) 12

(d) 10

Ans. (d) 10

18. The perimeter of an equilateral triangle is 21 yard. what is the length of its each sides

(a) 7 yard

(b) 14 yard

(c) 8 yard

(d) 12 yard

Ans. (a) 7 yard

19. What is the area of a triangle with base 12 m and a height of 18 m

(a) $208m^2$

(b) $126m^2$

(c) $108m^2$

(d) $98m^2$

Ans. (c) $108m^2$

20. Find the area of parallelogram if base = 8 and corresponding Altitude = 4

(a) 12

(b) 32

(c) 4

(d) 8

Ans. (b) 32

CBSE Class 9 Mathemaics

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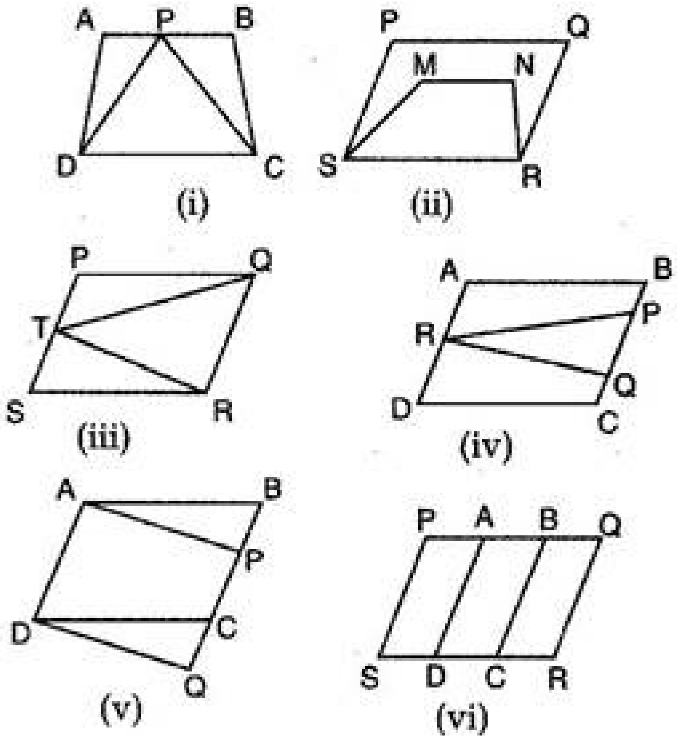
2 Marks Questions

1. Which of the following figures lie on the same base and between the same parallels. In such a case, write the common base and the two parallels.



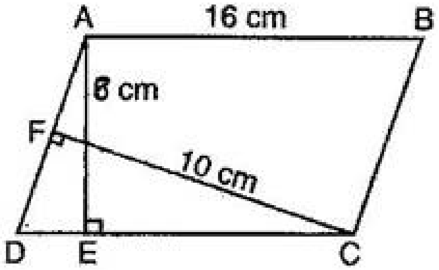
Ans. In figure (i): ΔDPC and trap. ABCD are on the same base DC and between same parallel DC and AB.

In figure (iii): ΔRTQ and parallelogram PQRS are on the same base QR and between same parallel QR and PS.

In figure (v): Parallelogram ABCD and parallelogram APQD are on the same base AD and between the same parallels AD and BQ.

2. In figure, ABCD is a parallelogram. AE $\perp$ DC and CF $\perp$ AD. If AB = 16 cm, AE = 8 cm

and CF = 10 cm, find AD.



Ans. ABCD is a parallelogram.

$\therefore$ DC = AB $\Rightarrow$ DC = 16 cm

AE $\perp$ DC[Given]

Now Area of parallelogram ABCD = Base $\times$ Corresponding height

$= DC \times AE = 16 \times 8 = 128\ cm^2$

Using base AD and height CF, we can find,

Area of parallelogram $= AD \times CF$

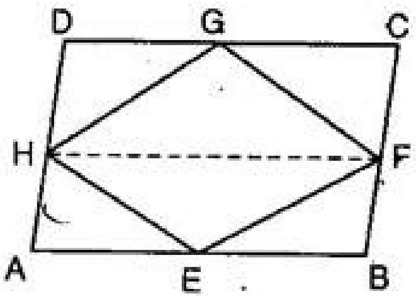
$\Rightarrow 128 = AD \times 10$

$\Rightarrow$ AD = $\frac{128}{10}$ = 12.8 cm

3. If E, F, G and H are respectively the mid-points of the sides of a parallelogram ABCD, show that ar (EFGH) = $\frac{1}{2}$ ar (ABCD).

Ans. Given: A parallelogram ABCD. E, F, G and H are mid-points of AB,

BC, CD and DA respectively.



To prove: ar (EFGH) = $\frac{1}{2}$ ar (ABCD)

Construction: Join HF

Proof: ar (Δ GHF) = $\frac{1}{2}$ ar ($\parallel$ gm HFCD)..........(i)

And ar (Δ HEF) = $\frac{1}{2}$ ar ($\parallel$ gm HABF)..........(ii)

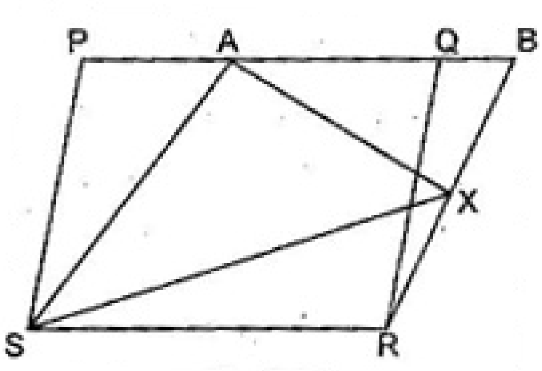
[If a triangle and a parallelogram are on the same base and between the same parallel then the area of triangle is half of area of parallelogram]

Adding eq. (i) and (ii),

ar (Δ GHF) + ar (Δ HEF) = $\frac{1}{2}$ ar ($\parallel$ gm HFCD) + $\frac{1}{2}$ ar ($\parallel$ gm HABF)

$\Rightarrow$ ar ($\parallel$ gm HEFG) = $\frac{1}{2}$ ar ($\parallel$ gm ABCD)

4. In figure, PQRS and ABRS are parallelograms and X is any point on the side BR. Show that:



(i) ar (PQRS) = ar (ABRS)

(ii) ar (AXS) = $\frac{1}{2}$ ar (PQRS)

Ans. (i) Parallelogram PQRS and ABRS are on the same base SR and between the same parallels SR and PB.

$\therefore$ ar (|| gm PQRS) = $\frac{1}{2}$ ar (|| gm ABRS)……….(i)

[$\because$ parallelograms on the same base and between the same parallels are equal in area]

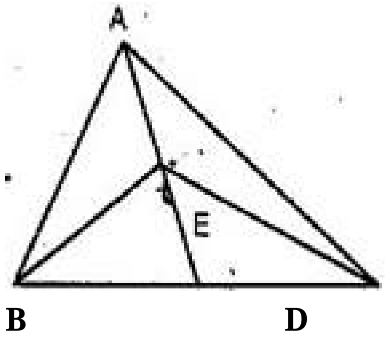
(ii) $\triangle$AXS and || gm ABRS are on the same base AS and between the same parallels AS and BR.

$\therefore$ ar ($\triangle$AXS) = $\frac{1}{2}$ ar (|| gm ABRS)……….(ii)

Using eq. (i) and (ii),

ar ($\triangle$AXS) = $\frac{1}{2}$ ar (|| gm PQRS)

5. In figure, E is any point on median AD of a $\triangle$ABC. Show that ar ($\triangle$ABE) = ar ($\triangle$ACE).



Ans. In $\triangle$ABC, AD is a median.

ar ($\triangle$ABD) = ar ($\triangle$ACD) ……….(i)

[$\because$ Median divides a $\triangle$ into two $\triangle$s of equal area]

Again in $\triangle$EBC, ED is a median

ar ($\triangle$EBD) = ar ($\triangle$ECD)……….(ii)

Subtracting eq. (ii) from (i),

ar ($\triangle$ABD) – ar ($\triangle$EBD) = ar ($\triangle$ACD) – ar ($\triangle$ECD)

$\Rightarrow$ ar ($\triangle$ABE) = ar ($\triangle$ACE)

6. **Show that DE||BC if ar $(\Delta BCE) = $ ar (ΔBCD)**

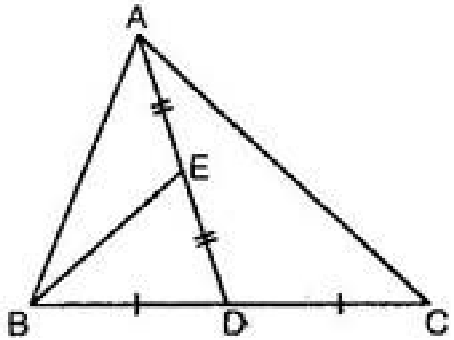
Ans. Since Δs BCE and BCD are equal in area and have a same base BC

Δ BCE and Δ BCD are between the same Parallel lines.

DE||BC

7. **In a triangle ABC, E is the mid-point of median AD. Show that ar (BED) = $\frac{1}{4}$ ar (ABC).**

Ans. Given: A Δ ABC, AD is the median and E is the mid-point of median AD.



To prove: ar (Δ BED) = $\frac{1}{4}$ ar (Δ ABC)

Proof: In Δ ABC, AD is the median.

$\therefore$ ar (Δ ABD) = ar (Δ ADC)

[$\because$ Median divides a Δ into two Δ s of equal area]

$\Rightarrow$ ar (Δ ABD) = $\frac{1}{2}$ ar (ABC)..........(i)

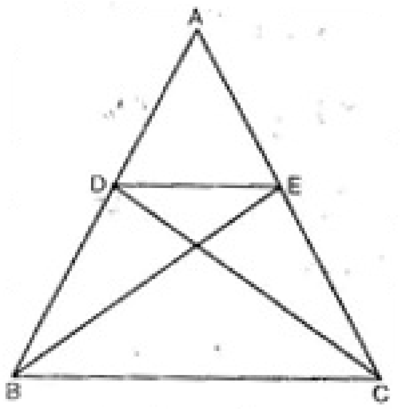
In Δ ABD, BE is the median.

$\therefore$ ar (Δ BED) = ar (Δ BAE)

$\Rightarrow$ ar (Δ BED) = $\frac{1}{2}$ ar (Δ ABD)

$\Rightarrow$ ar ($\triangle$BED) = $\frac{1}{2} \times \frac{1}{2}$ ar ($\triangle$ABC) = $\frac{1}{4}$ ar ($\triangle$ABC)

8. D and E are points on sides AB and AC respectively of $\triangle$ABC such that ar (DBC) = ar (EBC). Prove that DE || BC.

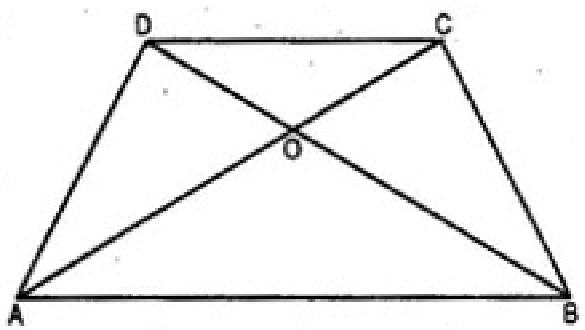


Ans. Given: ar ($\triangle$DBC) = ar ($\triangle$EBC)

Since two triangles of equal area have common base BC.

Therefore DE || BC[$\because$ Two triangles having same base (or equal bases) and equal areas lie between the same parallel]

9. Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at O. Prove that ar(AOD) = ar (BOC).



Ans. $\triangle$ABD and $\triangle$ABC lie on the same base AB and between the same parallels AB and DC.

$\therefore$ ar ($\triangle$ABD) = ar ($\triangle$ABC)

Subtracting ar ($\triangle$AOB) from both sides, ar ($\triangle$ABD) – ar ($\triangle$AOB)

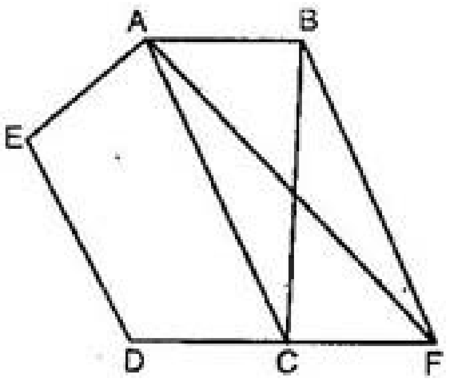
= ar ($\triangle$ABC) – ar ($\triangle$AOB)

$\Rightarrow$ ar ($\triangle$AOD) = ar ($\triangle$BOC)

10. In figure, ABCDE is a pentagon. A line through B parallel to AC meets DC produced at F. Show that:

(i)ar (ACB = ar (ACF)

(ii)ar (AEDF) = ar (ABCDE)



Ans. (i) Given that BF || AC

$\triangle$ACB and $\triangle$ACF lie on the same base AC and between the same parallels AC and BF.

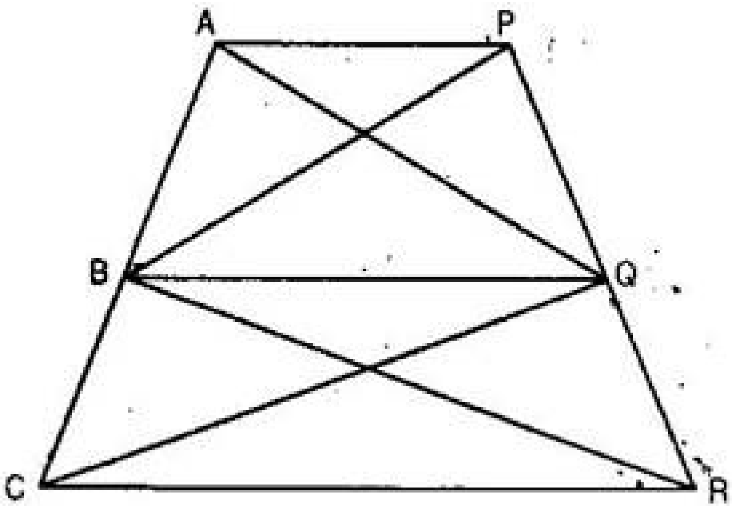
$\therefore$ ar ($\triangle$ACB) = ar ($\triangle$ACF)..........(i)

(ii) Nowar (ABCDE) = ar (trap. AEDC) + ar ($\triangle$ABC)..........(ii)

$\Rightarrow$ ar (ABCDE) = ar (trap. AEDC) + ar ($\triangle$ACF) = ar (quad. AEDF)[Using (i)]

$\Rightarrow$ ar (AEDF) = ar (ABCDE)

11. In figure, AP || BQ || CR. Prove that ar (AQC) = ar (PBR).



Ans. $\triangle$ABQ and BPQ lie on the same base BQ and between same parallels AP and BQ.

$\therefore$ ar ($\triangle$ABQ) = ar ($\triangle$BPQ)..........(i)

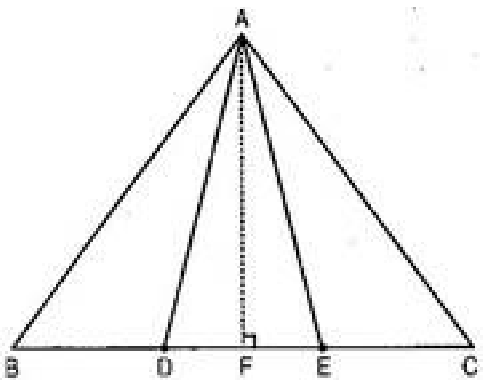
$\triangle$BQC and $\triangle$BQR lie on the same base BQ and between same parallels BQ and CR.

$\therefore$ ar ($\triangle$BQC) = ar ($\triangle$BQR)..........(ii)

Adding eq (i) and (ii),ar ($\triangle$ABQ) + ar ($\triangle$BQC) = ar ($\triangle$BPQ) + ar ($\triangle$BQR)

$\Rightarrow$ ar ($\triangle$AQC) = ar ($\triangle$PBR)

12. In figure, D and E are two points on BC such that BD = DE = EC. Show that ar (ABD) = ar (ADE) = ar (AEC). Can you know answer the question that you have left in the 'introduction' of this chapter, whether the field of Budhia has been actually divided into three parts of equal area?



Ans. In $\triangle$ABC, points D and E divides BC in three equal parts such that BD = DE = EC.

$\therefore$ BD = DE = EC = $\frac{1}{3}$ BC

Draw AF $\perp$ BC

ar (ΔABC) = $\frac{1}{2} \times BC \times AF$ ....(i)

and ar (ΔABD) = $\frac{1}{2} \times BD \times AF$ ....(ii)

= $\frac{1}{2} \times \frac{BC}{3} \times AF$ = $\frac{1}{3} \times \left[\frac{1}{2} \times BC \times AF\right]$

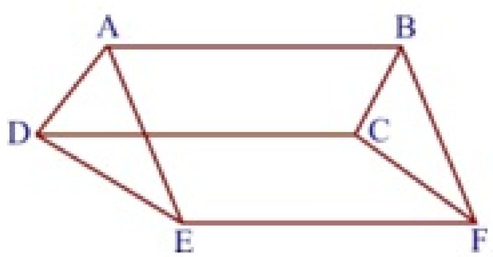
= $\frac{1}{3}$ ar (ΔABC)..........(iii)

And ar (ΔAEC) = $\frac{1}{3}$ ar (ΔABC)..........(iv)

From (ii), (iii) and (iv),

ar (ΔABD) = ar (ΔADE) = ar (ΔAEC)

13. In figure, ABCD, DCFE and ABFE are parallelograms. Show that ar (ADE) = ar (BCF).



Ans. As we know that opposite sides of a parallelogram are always equal.

$\therefore$ In parallelogram ABFE,AE = BF and AB = EF

In parallelogram DCFE,DE = CF and DC = EF

In parallelogram ABCD,AD = BC and AB = DC

Now in ΔADE and ΔBCF,

AE = BF[Opposite sides of parallelogram ABFE]

DE = CF[Opposite sides of parallelogram DCFE]

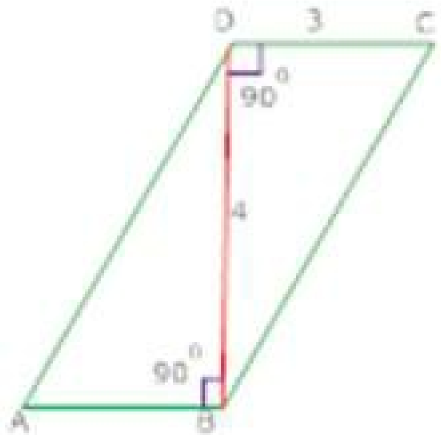
And AD = BC[Opposite sides of parallelogram ABCD]

$\therefore$ ΔADE $\cong$ ΔBCF[By SSS congruency]

$\therefore$ ar (ΔADE) = ar (ΔBCF)

[$\because$ Area of two congruent figures is always equal]

14. Prove that ABCD is a parallelogram. If ABCD is a quadrilateral and BD is one of its diagonal.



Ans. Given quadrilateral ABCD in which AB = DC = 3, BD = 4 and $\angle AB = \angle BDC = 90°$

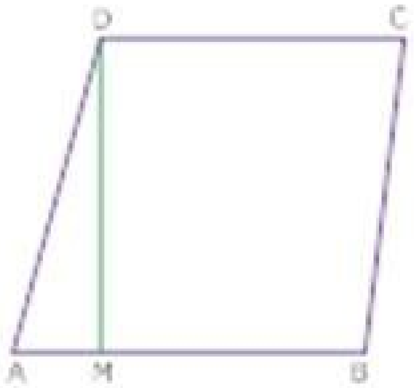
BD intersects AB and DC such that.

$\angle ABD = \angle BDC = 90°$
$\therefore AB \parallel CD$ (alternate interior angles are equal)
but $AB = DC = 3$

Thus, ABCD is a parallelogram.

15. In a parallelogram ABCD, AB= 20. The altitude DM to sides AB is 10 cm. Find area of

parallelogram.



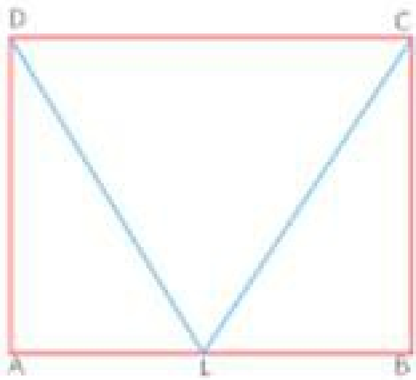
Ans. Area of parallelogram ABCD

= AB$\times$DM

=20$\times$10

=200 square cm.

16. If L be any Point on AB and the area of rectangle ABCD is 100 square cm. find area of ΔLCD.



Ans. Area of rectangle ABCD = 100 square cm

Area $\Delta LCD = \frac{1}{2}$ area rectangle ABCD

= $\frac{1}{2} \times 100$ square cm

=50 square cm

17. Find the area of parallelogram ABCD, BD is perpendicular on AB. AB = 7 and BD is 5.

Ans. Area of parallelogram = Base $\times$ Corresponding Altitude $=7\times 5$

= 35 square cm

Area of parallelogram = 35 square cm

18. Show that ar (ABC) = ar (ABD). ABC and ABD are two triangles on the same base AB if line segment CD is bisected by AO at O

Ans. AO is the median of ΔACD

$ar(\Delta AOC) = ar(\Delta AOD)$

$ar(\Delta BOC) = ar(\Delta BOD)$

$ar(\Delta AOC) + ar(\Delta BOC) = ar(\Delta AOD) + ar(\Delta BOD)$

$ar(\Delta ABC) = ar(\Delta ABD)$

19. Show that BDEF is parallelogram. If D, E and F the mid- points of the side BC, CA and AB of triangle ABC

Ans. Join DE, EF and FD

E and F are the mid-points of AC and AB

EF|| BC

EF || BD

DE|| BF

BDEF is a || gram.

20. Prove that ar (ΔOLP)= ar (ΔMNL) if MN||PO

Ans. $ar(\Delta MPO) = ar(\Delta MPN)$

$ar(\Delta MPO) - ar(\Delta MPL) = ar(\Delta MPN) - ar(\Delta MPL)$

$ar(\Delta OLP) = ar(\Delta MLN)$

21. Justify the line corresponding to side EF if ar $(\Delta ABC) = ar(\Delta DEF)$ in $\Delta ABC, AB = 8$ and altitude AB is 5 cm and $\Delta DEF, EF = 10cm$

Ans. Given that $ar(ABC) = ar(\Delta DEF)$

$\frac{1}{2} \times AB \times AM = \frac{1}{2} \times EF \times DN$

$\frac{1}{2} \times 8 \times 5 = \frac{1}{2} \times 10 \times DN$

$20 = 5DN$

$DN = 4cm$

22. In a parallelogram PQRS, PQ = 6 cm and the corresponding altitude ST is 5 cm. find area of parallelogram.

Ans. Area of ||gm PQRS

= Base $\times$ Altitude

= 6×5 (Square cm)

= 30 square cm

23. Show that the median of a triangle divides it into two triangles of equal area.

Ans. Given: A triangle PQR and PS is the median

To prove: ar (PQS) = ar (PSR)

Construction: Draw the altitude PT from vertex P on the base QR

Proof: Area of $\Delta PQS = \frac{1}{2} \times QS \times PT$

And Area of $\Delta PSR = \frac{1}{2} \times SR \times PT$

$= \frac{1}{2} \times QS \times PT \left[\because QS = SR \right]$

$\therefore$ area of ΔPQS of ΔPSR

Hence proved.

24. The area of rectangle PQRS is 500 sq cm. if T be any Point on PQ, find area of ΔTRS.

Ans. As of $\Delta TRS = \frac{1}{2}$ as rectangle PQRS

$= \frac{1}{2} \times 500$ Square cm

=250 square cm

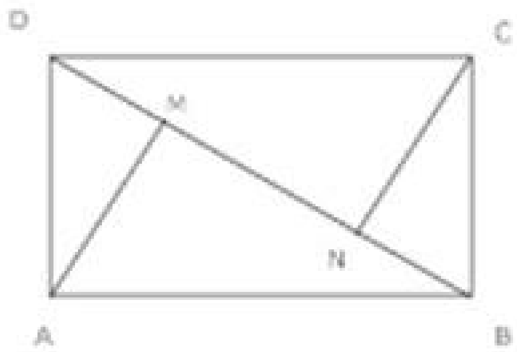
25. Prove that as $(\Delta ROS) =$ ar (ΔPQO) if PS||RQ

Ans. $ar(\Delta PSR) = ar(\Delta PSQ)$

$ar(\Delta PSR) - ar(\Delta PSO) = ar(\Delta PSQ) - ar(\Delta PSO)$

$ar(\Delta ROS) = ar(\Delta PQO)$

26. Show that ar (quad. ABCD)= $\frac{1}{2}$ BD (AM+CN) BD is one of the diagonals of a quadrilateral ABCD, AM and CN are the $\perp$ from A and C

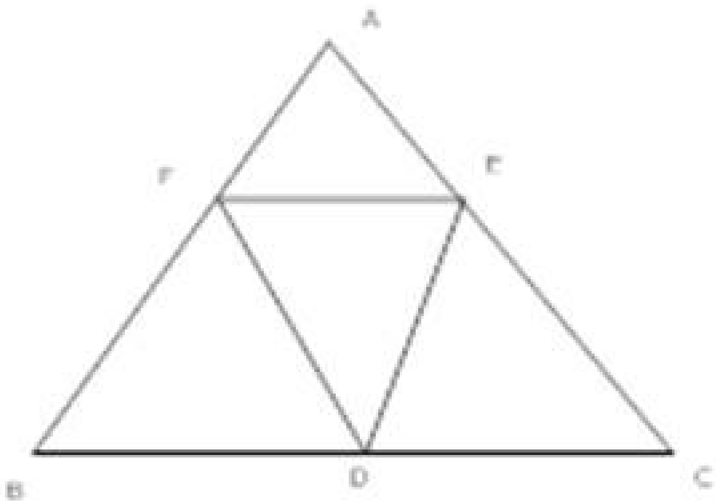


Ans. $\text{ar}(\text{quad.}ABCD) = ar(\Delta ABD) + ar(\Delta BCD)$

$= \frac{1}{2}(BD \times AM) + \frac{1}{2}(BD \times CN)$

$= \frac{1}{2}BD(AM + CN)$

27. D, E, F are respectively the mid-points of the sides BC, CA and AB of ΔABC Prove that ar ($\triangle$ DEF) = $\frac{1}{4}$ ar ($\triangle$ ABC).



Ans. $ar(\Delta BDF) = ar(\Delta DEF)$

Now, $ar(\| gram BDEF) = 2ar(\Delta DEF)$

$= 2 \times \frac{1}{4} ar(\Delta ABC)$

$= \frac{1}{2} ar(\Delta ABC)$

28. In a parallelogram PQRS, PS = 12. The altitude to side PS is equal to 12cm. find area of parallelogram PQRS

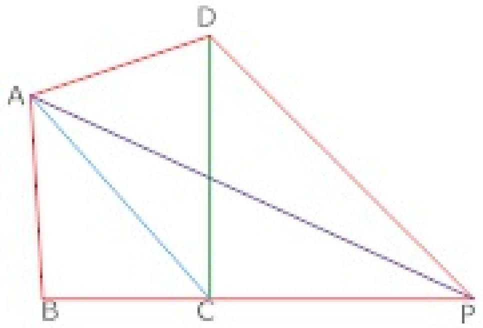
Ans. Area of parallelogram PQRS

=Base $\times$ Corresponding Altitude

=12 $\times$ 12

=144 square cm

29. A line through D, Parallel to AC meets BC produced in P. prove that area $\Delta ABP = ar ABCD$.



Ans. $ar(\Delta ACP) = ar(\Delta ACD)$

$ar(\Delta ACP) + ar(\Delta ABC) = ar(\Delta ACD) + ar(\Delta ABC)$

$ar(\Delta ABP) = ar(\text{quad.} ABCD)$

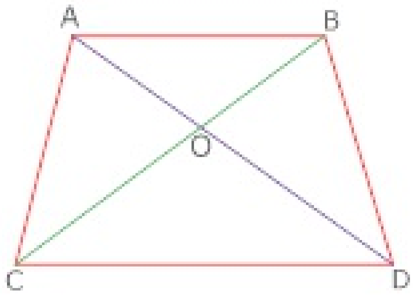
30. In a parallelogram PQRS, PQ = 13. The altitude corresponding to sides PQ is equal to 5 cm. find the area of parallelogram.

Ans. Area of parallelogram = base $\times$ Altitude

=13$\times$5

=65 cm

31. Prove that ar (AOD) = ar (BOC). Diagonals AC and BD of a trapezium ABCD with AB||DC intersect each other at O.

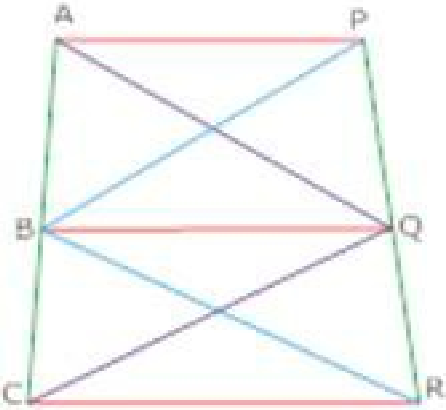


Ans. Ar$\left(\Delta ADC\right) = ar\left(\Delta BDC\right)$

$ar\left(\Delta ADC\right) - ar\left(\Delta ODC\right) = ar\left(\Delta BDC\right) - ar\left(\Delta ODC\right)$

$ar\left(\Delta AOC\right) = ar\left(\Delta BOC\right)$

32. Prove that ar (AQC) = ar (PBR) if AP||BQ||CR.



Ans. $ar\left(\Delta ABQ\right) = ar\left(\Delta PBQ\right)$

$ar\left(\Delta CBQ\right) = ar\left(\Delta RBQ\right)$

$ar\left(\Delta ABQ\right) + ar\left(\Delta CBQ\right) = ar\left(\Delta PBQ\right) + ar\left(\Delta PBQ\right)$

$ar\left(AQC\right) = ar\left(PBR\right)$

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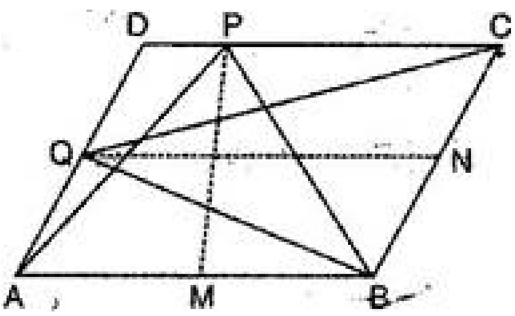
3 Marks Questions

1. P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD. Show that ar (APB) = ar (BQC).

Ans. Given: ABCD is a parallelogram. P is a point on DC and Q is a point on AD.



To prove: ar ($\triangle$APB) = ar ($\triangle$BQC)

Construction: Draw PM $\parallel$ BC and QN $\parallel$ DC.

Proof: Since QC is the diagonal of parallelogram QNCD.

$\therefore$ ar ($\triangle$QNC) = $\frac{1}{2}$ ar ($\parallel$gm QNCD) ....(i)

Again BQ is the diagonal of parallelogram ABNQ.

$\therefore$ ar ($\triangle$BQN) = $\frac{1}{2}$ ar ($\parallel$gm ABNQ) ....(ii)

Adding eq. (i) and (ii),

ar ($\triangle$QNC) + ar ($\triangle$BQN) = $\frac{1}{2}$ ar ($\parallel$gm QNCD) + $\frac{1}{2}$ ar ($\parallel$gm ABNQ)

$\Rightarrow$ ar ($\triangle$BQC) = $\frac{1}{2}$ ar ($\|$gm ABCD) ....(iii)

Again AP is the diagonal of $\|$gm AMPD.

$\therefore$ ar ($\triangle$APM) = $\frac{1}{2}$ ar ($\|$gm AMPD) ....(iv)

And PB is the diagonal of $\|$gm PCBM.

$\therefore$ ar ($\triangle$PBM) = $\frac{1}{2}$ ar ($\|$gm PCBM) ....(v)

Adding eq. (iv) and (v),

ar ($\triangle$APM) + ar ($\triangle$PBM) = $\frac{1}{2}$ ar ($\|$gm AMPD) + $\frac{1}{2}$ ar ($\|$gm PCBM)

$\Rightarrow$ ar ($\triangle$APB) = $\frac{1}{2}$ ar ($\|$gm ABCD) ....(vi)

From eq. (iii) and (vi),

ar ($\triangle$BQC) = ar ($\triangle$APB) or ar ($\triangle$APB) = ar ($\triangle$BQC)

2. A farmer was having a field in the form of a parallelogram PQRS. She took any point A on RS and joined it to points P and Q. In how many parts the field is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portions of the field separately. How should she do it?

Ans. When A is joined with P and Q; the field is divided into three parts viz. $\triangle$PAS, $\triangle$APQ and $\triangle$AQR.

$\triangle$APQ and parallelogram PQRS are on the same base PQ and between same parallels PQ and SR.

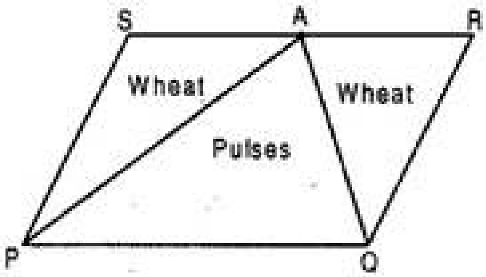
$\therefore$ ar ($\triangle$APQ) = $\frac{1}{2}$ ar ($\|$gm PQRS)

It implies that triangular region APQ covers half portion of parallelogram shaped field PQRS.

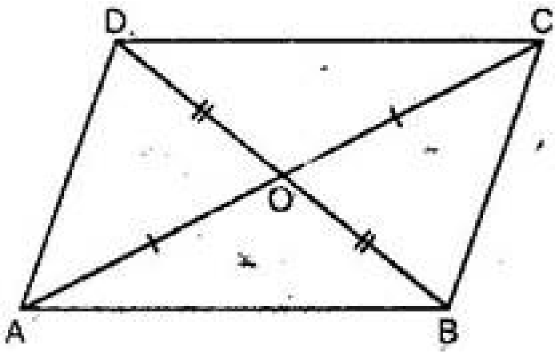
So if farmer sows wheat in triangular shaped field APQ then she will definitely sow pulses in other two triangular parts PAS and AQR.

Or

When she sows pulses in triangular shaped field APQ then she will sow wheat in other two triangular parts PAS and AQR.



3. Show that the diagonals of a parallelogram divide it into four triangles of equal area.



Ans. Let parallelogram be ABCD and its diagonals AC and BD intersect each other at O.

In $\triangle ABC$ and $\triangle ADC$,

AB = DC [Opposite sides of a parallelogram]

BC = AD [Opposite sides of a parallelogram]

And AC = AC [Common]

$\therefore$ $\triangle ABC \cong \triangle CDA$ [By SSS congruency]

Since, diagonals of a parallelogram bisect each other.

∴ O is the mid-point of bisection.

Now in $\triangle$ADC, DO is the median.

∴ ar ($\triangle$AOD) = ar ($\triangle$COD) ....(i)

[Median divides a triangle into two equal areas]

Similarly, in $\triangle$ABC, OB is the median.

∴ ar ($\triangle$AOB) = ar ($\triangle$BOC) ....(ii)

And in $\triangle$AOB and $\triangle$AOD, AO is the median.

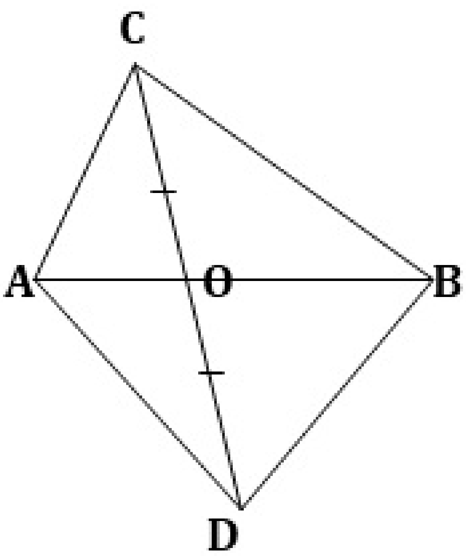
∴ ar ($\triangle$AOB) = ar ($\triangle$AOD) ....(iii)

From eq. (i), (ii) and (iii),

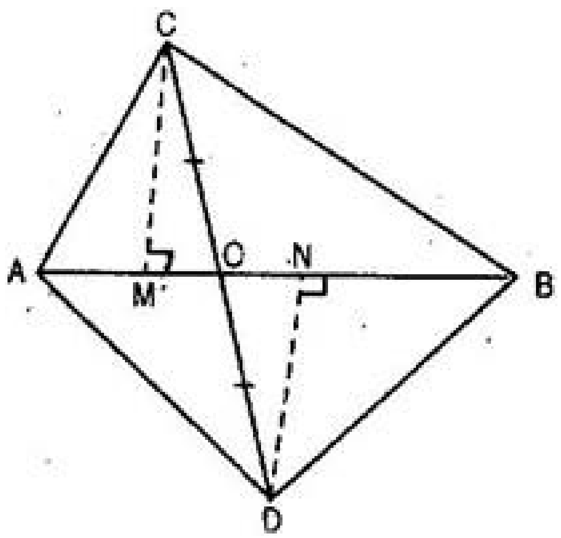
ar ($\triangle$AOB) = ar ($\triangle$AOD) = ar ($\triangle$BOC) = ar ($\triangle$COD)

Thus diagonals of parallelogram divide it into four triangles of equal area.

4 In figure, ABC and ABD are two triangles on the same base AB. If line-segment CD is bisected by AB at O, show that ar (ABC) = ar (ABD).



Ans. Draw CM $\perp$ AB and DN $\perp$ AB.



In $\triangle$CMO and $\triangle$DNO,

$\angle$CMO = $\angle$DNO = 90° [By construction]

$\angle$COM = $\angle$DON [Vertically opposite]

OC = OD [Given]

$\therefore$ $\triangle$CMO $\cong$ $\triangle$DNO [By ASA congruency]

$\because$ AM = DN [By CPCT] ……(i)

Now ar ($\triangle$ABC) = $\frac{1}{2}\times AB\times CM$ ……….(ii)

ar ($\triangle$ADB) = $\frac{1}{2}\times AB\times DN$ ……….(iii)

Using eq. (i) and (iii),

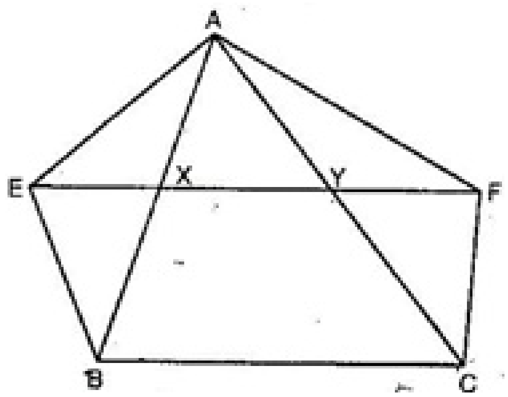
ar ($\triangle$ADB) = $\frac{1}{2}\times AB\times CM$ ……….(iv)

From eq. (ii) and (iv),

ar ($\triangle$ABC) = ar ($\triangle$ADB)

5. XY is a line parallel to side BC of triangle ABC. If BE $\parallel$ AC and CF $\parallel$ AB meet XY at E and

F respectively, show that ar (ABE) = ar (ACF).



Ans. $\triangle$ABE and parallelogram BCYE lie on the same base BE and between the same parallels BE and AC.

$\therefore$ ar ($\triangle$ABE) = $\frac{1}{2}$ ar ($\|$ gm BCYE) ....(i)

Also $\triangle$ACF and $\|$ gm BCFX lie on the same base CF and between same parallel BX and CF.

$\therefore$ ar ($\triangle$ACF) = $\frac{1}{2}$ ar ($\|$ gm BCFX) ....(ii)

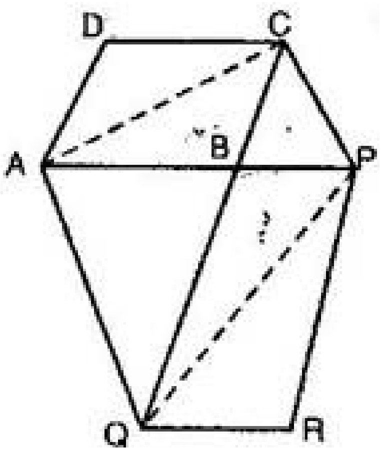
But $\|$ gm BCYE and $\|$ gm BCFX lie on the same base BC and between the same parallels BC and EF.

$\therefore$ ar ($\|$ gm BCYE) = ar ($\|$ gm BCFX) ....(iii)

From eq. (i), (ii) and (iii), we get,

ar ($\triangle$ABE) = ar ($\triangle$ACF)

6. The side AB of parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed. Show that ar (ABCD) = ar (PBQR).



Ans. Given: ABCD is a parallelogram, CP || AQ and PBQR is a parallelogram.

To prove: ar (ABCD) = ar (PBQR)

Construction: Join AC and QP.

Proof: Since AQ || CP

$\therefore$ ar (ΔAQC) = ar (ΔAQP)

[Triangles on the same base and between the same parallels are equal in area]

Subtracting ar (ΔABQ) from both sides, we get

ar (ΔAQC – ar (ΔABQ) = ar (ΔAQP) – ar (ΔABQ)

$\Rightarrow$ ar (ΔABC) = ar (ΔQBP) ....(i)

Now ar (ΔABC) = $\frac{1}{2}$ ar (|| gm ABCD)

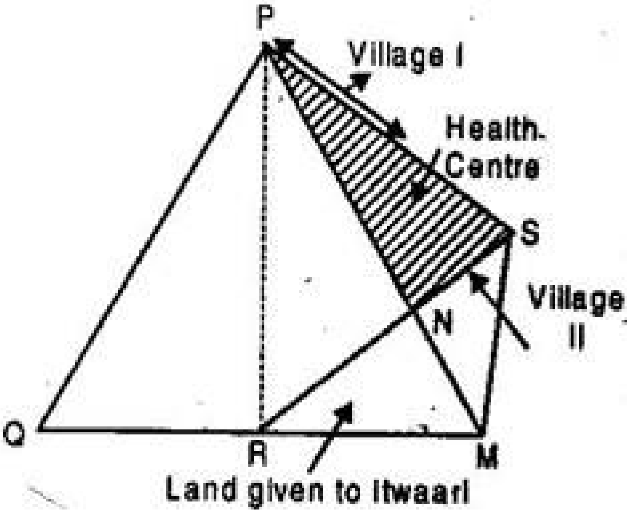
[Diagonal divides a parallelogram in two parts of equal area]

And ar (ΔPQB) = $\frac{1}{2}$ ar (|| gm PBQR)

From eq. (i), (ii) and (iii), we get

ar (|| gm ABCD) = ar (|| gm PBQR)

7. A villager Itwaari has a plot of land of the shape of quadrilateral. The Gram Panchyat of two villages decided to take over some portion of his plot from one of the corners to construct a health centre. Itwaari agrees to the above personal with the condition that he should be given equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.



Ans. Let Itwaari has land in shape of quadrilateral PQRS.

Draw a line through 5 parallel to PR, which meets QR produced at M.

Let diagonals PM and RS of new formed quadrilateral intersect each other at point N.

We have PR || SM [By construction]

$\therefore$ ar ($\triangle$PRS) = ar ($\triangle$PMR)

[Triangles on the same base and same parallel are equal in area]

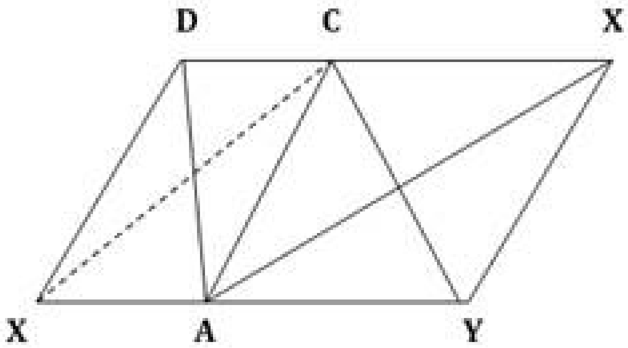
Subtracting ar ($\triangle$PNR) from both sides,

ar ($\triangle$PRS) – ar ($\triangle$PNR) = ar ($\triangle$PMR) – ar ($\triangle$PNR)

$\Rightarrow$ ar ($\triangle$PSN) = ar ($\triangle$MNR)

It implies that Itwari will give corner triangular shaped plot PSN to the Grampanchayat for health centre and will take equal amount of land (denoted by $\triangle$MNR) adjoining his plot so as to form a triangular plot PQM.

8. ABCD is a trapezium with AB || DC. A line parallel to AC intersects AB at X and BC at Y. Prove that ar (ADX) = ar (ACY).



Ans. Join CX, $\triangle$ADX and ACX lie on the same base

XA and between the same parallels XA and DC.

$\therefore$ ar ($\triangle$ADX) = ar ($\triangle$ACX) ....(i)

Also $\triangle$ACX and $\triangle$ACY lie on the same base

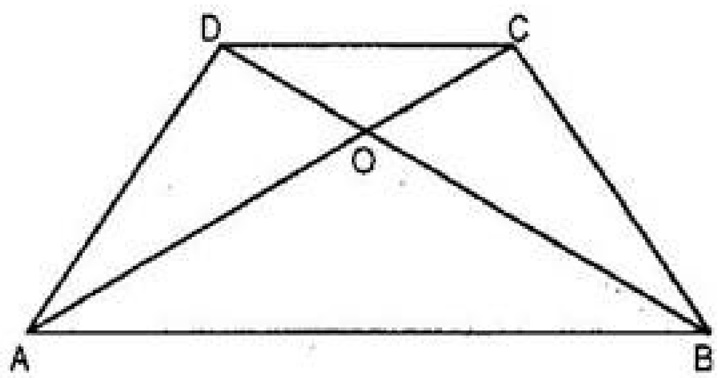
AC and between same parallels CY and XA.

$\therefore$ ar ($\triangle$ACX) = ar ($\triangle$ACY) ....(ii)

From (i) and (ii),

ar ($\triangle$ADX) = ar ($\triangle$ACY)

9. Diagonals AC and BD of a quadrilateral ABCD intersect at O in such a way that ar (AOD) = ar(BOC). Prove that ABCD is a trapezium.



Ans. Given that ar ($\triangle$AOD) = ar ($\triangle$BOC)

Adding $\triangle$AOB both sides,

ar ($\triangle$AOD) + ar ($\triangle$AOB) = ar ($\triangle$BOC) + ar ($\triangle$AOB)

$\Rightarrow$ ar ($\triangle$ABD) = ar ($\triangle$ABC)

Since if two triangles equal in area, lie on the same base then, they lie between same parallels. We have $\triangle$ABD and $\triangle$ABC lie on common base AB and are equal in area.

$\therefore$ They lie in same parallels AB and DC.

$\Rightarrow$ AB $\parallel$ DC

Now in quadrilateral ABCD, we have AB $\parallel$ DC

Therefore ABCD is trapezium.[$\because$ In trapezium one pair of opposite sides is parallel]

10. In figure, ar (DRC) = ar (DPC) and ar (BDP) = ar (ARC). Show that both the quadrilaterals ABCD and DCPR are trapeziums.

Ans. Given that $\triangle$DRC and $\triangle$DPC lie on the same base DC and ar ($\triangle$DPC) = ar ($\triangle$DRC) .....(i)

$\therefore$ DC $\parallel$ RP

[If two triangles equal in area, lie on the same base then, they lie between same parallels]

Therefore, DCPR is trapezium. [$\because$ In trapezium one pair of opposite sides is parallel]

Also ar ($\triangle$BDP) = ar ($\triangle$ARC) ....(ii)

Subtracting eq. (i) from (ii),

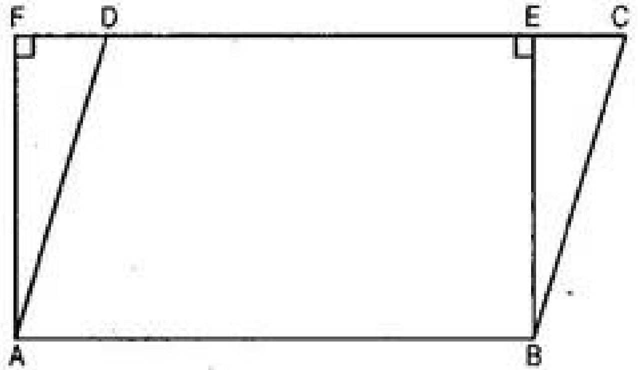
ar ($\triangle$BDP) – ar ($\triangle$DPC) = ar ($\triangle$ARC) – ar ($\triangle$DRC)

$\Rightarrow$ ar ($\triangle$BDC) = ar ($\triangle$ADC)

Therefore, AB $\parallel$ DC [If two triangles equal in area, lie on the same base then, they lie between same parallels]

Therefore, ABCD is trapezium.

11. Parallelogram ABCD and rectangle ABEF are on the same base AB and have equal areas. Show that the perimeter of the parallelogram is greater than that of the rectangle.



Ans. Given: Parallelogram ABCD and rectangle ABEF are on same base AB and between the same parallels AB and CF.

$\therefore$ ar (||gm ABCD) = ar (rect. ABEF)

To prove: AB + BC + CD + AD > AB + BE + EF + AF

Proof: AB = CD [$\because$ opposites sides of a

parallelogram are always equal]

AB = EF [$\because$ opposites sides of a

rectangle are always equal]

$\therefore$ CD = EF

Adding AB both sides,

AB + CD = AB + EF ....(i)

$\therefore$ Off all the segments that can be drawn to a given line from a point not lying on it, the perpendicular segment is the shortest.

$\therefore$ BE < BC and AF < AD

$\Rightarrow$ BC > BE and AD > AF

$\therefore$ BC + AD > BE + AF ....(ii)

From eq. (i) and (ii),

AB + CD + BC + AD = AB + EF + BE + AF

12. In figure, ABCD is a parallelogram and BC is produced to a point Q such that AD = CQ. If AQ intersects DC at P, show that ar (BPC) = ar (DPQ).

Ans. Join A and C.

$\triangle$APC and $\triangle$BPC are on the same base PC and between the same parallels PC and AB.

$\therefore$ ar ($\triangle$APC) = ar ($\triangle$BPC) ....(i)

Now ACBD is a parallelogram.

AD = BC [opposite sides of a parallelogram are always equal]

Also BC = CQ [given]

$\therefore$ AD = CQ

Now AD $\parallel$ CQ [Since CQ is the extension of BC]

And AD = CQ

$\therefore$ ADQC is a parallelogram.

[$\because$ If one pair of opposite sides of a quadrilateral is equal and parallel then it is a parallelogram]

Since diagonals of a parallelogram bisect each other.

$\therefore$ AP = PQ and CP = DP

Now in $\triangle$APC and $\triangle$DPQ,

AP = PQ [Proved above]

$\angle$APC = $\angle$DPQ [Vertically opposite angles]

PC = PD [Prove above]

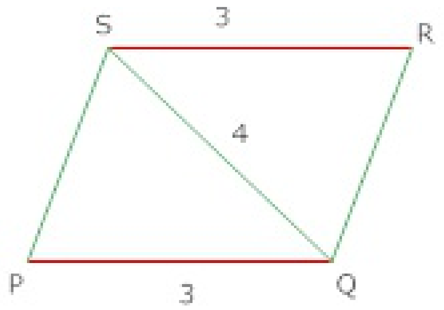
$\therefore$ ΔAPC $\cong$ ΔDPQ ....(ii)

$\Rightarrow$ ar (ΔAPC) = ar (ΔDPQ) [area of congruent figures is always equal]

From eq. (i) and (ii),

ar (ΔBPC) = ar (ΔDPQ)

13. PQRS is a quadrilateral and SQ is one of its diagonals. Show that PQRS is a Parallelogram and find its area too.



Ans. We know that, area of ||gram PQRS. In which

PQ=SR=3, SQ=4

And $\angle S = \angle Q = 90°$

$\angle PQS = \angle QSR = 90°$

PQ||SR

PQ=SR=3

ABCD is a ||gram

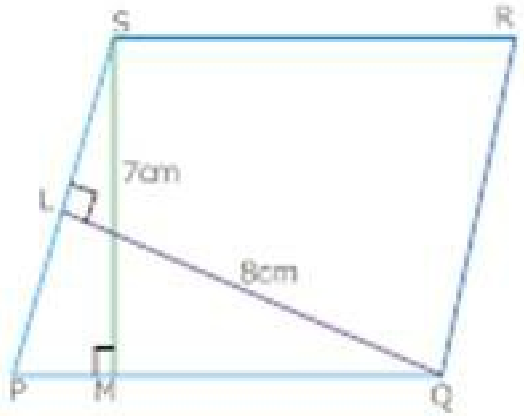
Area of parallelogram

=Base$\times$ corresponding Altitude

=3$\times$4

=12 square units

14. In a parallelogram PQRS. The Altitude corresponding to sides PQ and PS are respectively. 7 cm and 8 cm find PS, if PQ=10 cm.



Ans. Area of ||gram PQRS

=PQ$\times$SM

=10$\times$7

=70 square cm.............(i)

Area of Parallelogram PQRS

=PS$\times$QL

=(AD$\times$8) square cm...............(ii)

From (i) and (ii)

PS$\times$8=70

PS= $\frac{70}{8}$

=8.75 cm

15. Area, base and corresponding altitude are $x^2, x-3$ **and** $x+4$ **respectively. Find the**

area of parallelogram.

Ans. Area of parallelogram

= Base × Corresponding Altitude

$x^2 = (x-3)(x+4)$

$x^2 = x^2 + 4x - 3x - 12$

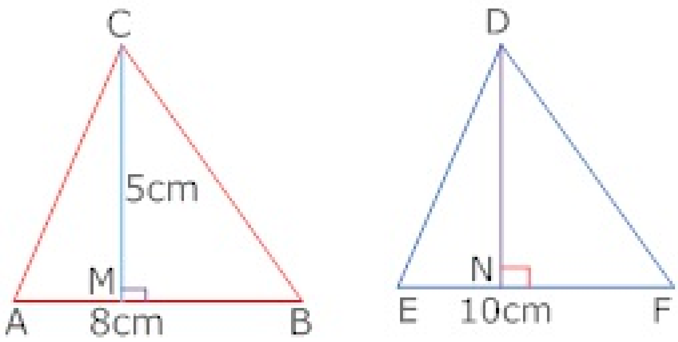
x=12

= (12-3)(12-4)

= (9)(16)

= 144 square units.

16. Find the altitude corresponding to side EF if area of $\Delta ABC = \Delta DEF$. If ΔABC AB = 8 cm and altitude corresponding to AB is 5 cm. In ΔDEF , $EF = 10\ cm$



Ans. $ar(\Delta ABC) = ar(\Delta DEF)$

$\frac{1}{2} \times AB \times CM = \frac{1}{2} \times EF \times DN$

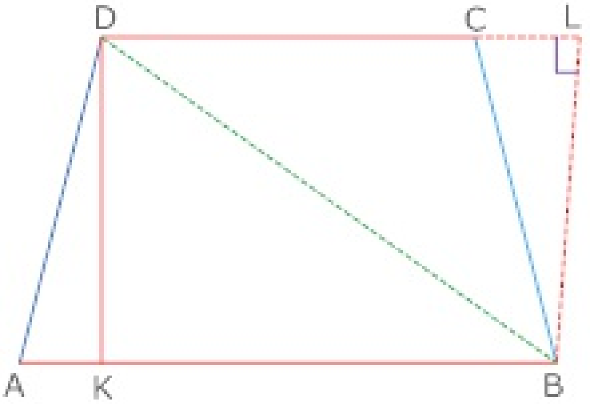
$\frac{1}{2} \times 8 \times 5 = \frac{1}{2} \times 10 \times DN$

$20 = 5DN$

$DN = 4cm$

Altitude corresponding to side EF is 4 cm

17. Prove that the area of a trapezium is half of the product of its height and the sum of the parallel sides.



Ans. Join B and D. Draw BL ⊥ DC (Produced)

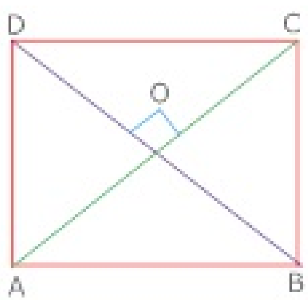
$ar(ABCD) = ar(\Delta ABD) + ar(\Delta DCB)$

$= \left(\frac{1}{2} AB \times DK\right) + \left(\frac{1}{2} DC \times BL\right)$

$= \left(\frac{1}{2} AB \times DK\right) + \left(\frac{1}{2} DC \times DK\right)$

$= \frac{1}{2} DK(AB + CD)$

18. Show that the area of a rhombus is half the product of the length of its diagonals.



Ans. $ar(\Delta ABC) = \frac{1}{2} \times AC \times OB$........(i)

$ar(\Delta ACD) = \frac{1}{2} \times AC \times DO$..........(ii)

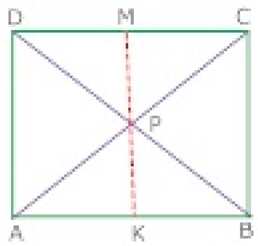
Adding (i) and (ii)

$ar(\Delta ABC + \Delta ACD)\frac{1}{2} \times AC \times (DO + OB)$

$= \frac{1}{2} \times AC \times BD$

Hence, area of rhombus ABCD = $\frac{1}{2} \times AC \times BD$

19. In parallelogram P is any point inside it. Prove that $ar(\Delta ABP) + ar\ (\Delta DCP) = \frac{1}{2} ar\ (\|\ gm\ ABCD)$



Ans. $ar(\Delta ABP) = \frac{1}{2} AB \times PK$

$ar(\Delta DCP) = \frac{1}{2} CD \times PM$

$= \frac{1}{2} AB \times PM$

$ar(\Delta ABP) + ar(\Delta DCP) = \frac{1}{2} AB \times PK + \frac{1}{2} AB \times PM$

$$= \frac{1}{2} AB(PK + PM)$$

$$= \frac{1}{2} AB \times MK$$

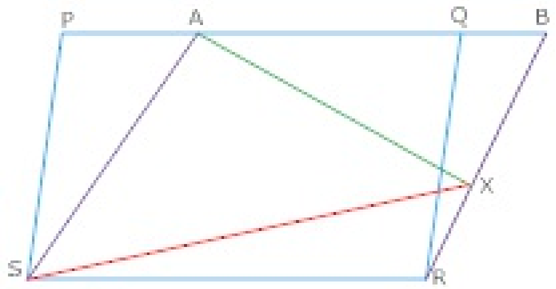
$$= \frac{1}{2} ar(\| \, gram \, ABCD)$$

20. Show that

(i) ar(PQRS) = ar (ABRS)

(ii) ar(AXS) = $\frac{1}{2}$ **ar(PQRS)**

If X **is any point on side BR of PQRS and ABRS.**



Ans. (i) ||gram PQRS and ABRS are on the same base SR and Between the same parallel PB and SR,

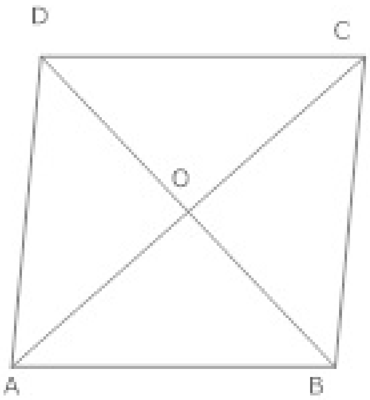
So, $ar(PQRS) = ar(ABRS)$

(ii) $ar(AXS) = \frac{1}{2} ar(ABRS)$

$ar(ABRS) = ar(PQRS)$

$ar(AXS) = \frac{1}{2} ar(PQRS)$

21. Show that the diagonals of a parallelogram divide if into four triangles of equal area.



Ans. Given: A parallelogram ABCD and AC and BC are diagonals

To prove: ar (ABO) = ar (COD) = ar (BCO) = ar (AOD)

Proof: ar (ADB) = ar (ACB)

$\Rightarrow ar(ADB) - ar(ABO) = ar(ACB) - ar(ABO)$

$\Rightarrow sr(ADO) = ar(BCO)$.......$(i)$

Ar (ADC) = ar (BCD)

$\Rightarrow ar(ADC) - ar(CDO) = ar(BCD) - ar(CDO)$

$\Rightarrow ar(ADO) = ar(AOB)$..........$(ii)$

In triangle ABC, BO is median

$\therefore ar(ABO) = ar(BCO)$........$(iii)$

In triangle ADC, OD is median

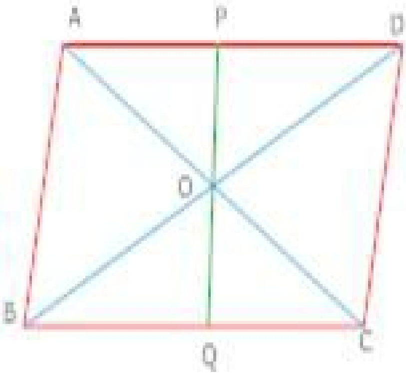
$\therefore ar(ADO) = ar(CDO)$.......$(iv)$

From (i), (ii), (iii) and (iv)

Ar(ABO) = ar (CDO) = ar (BCO) = ar (ADO)

Hence proved.

22. Show that PQ divides the ||gram in two Part of equal area if diagonal of ||gram ABCD intersect Point O. through Point O, a line is drawn to intersect AD at P and BC at Q.



Ans. To prove: Are (quadrilateral APQB) = ar (quadrilateral PQCD) = $\frac{1}{2}$ (ar ||gram ABCD)

Proof: $\angle AOP = \angle COQ$ (Vertically opposite angles)

$OA = OC$

$\angle OAP = \angle OCQ$

$\therefore \Delta AOP \cong \Delta DOQ$

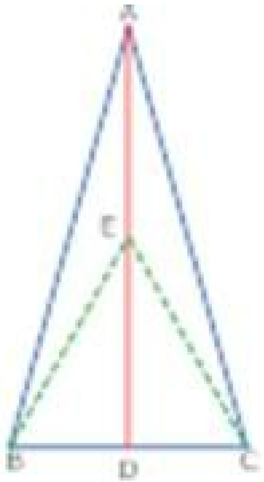
$\Rightarrow ar(\Delta AOP) = ar(\Delta COQ)$..........$(i)$

$ar(\Delta ABC) = ar(\Delta ACD)(\because$ Two triangles on the base and between same parallels$)$

$\Rightarrow$ ar (quad. ABQO) + $ar(\Delta COQ) = ar($quadrilateral OCDP$)+$ ar (ΔAOP)

$\Rightarrow$ ar (quad. APQB) = ar (quad. PQCD) $\left[\because ar(\Delta AOP) = ar(\Delta COQ)\right]$

23. Show that (ΔABE) = are of (ΔACE) if E is any Point on its median AD.



Ans. Join BE and CE

$ar(\Delta ABD) = ar(\Delta ACD)$ ....(i)

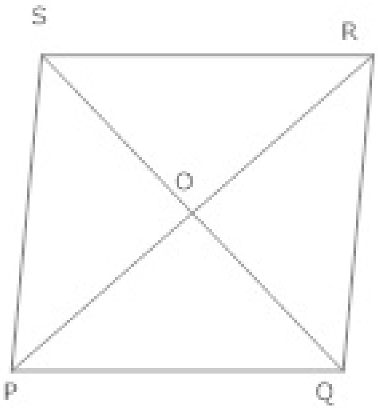
$ar(\Delta EBD) = ar(\Delta ECD)$...........$(ii)$

Subtracting (ii) from (i)

$ar(\Delta ABD) - ar(\Delta EBD) = ar(\Delta ACD) - ar(AECD)$

$\Rightarrow ar(\Delta ABE) = ar(\Delta ACE)$

24. The triangle PQR and PSR are equal in area, if PR and QS bisect at O.



Ans. PO is median of ΔPQS

$\therefore$ ar(ΔPOQ)=ar(ΔPOS)....(i)

RO is median of ΔQRS

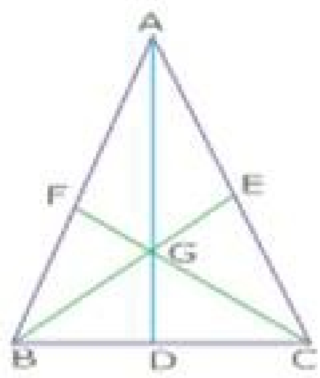
$\therefore$ ar $\left(\Delta QOR\right) = ar(\Delta ROS)..(ii)$

Adding (i) and (ii)

ar (ΔPOQ) + ar(ΔQOR) = ar $\left(\Delta POS\right) + ar\left(\Delta ROS\right)$

$\Rightarrow ar\left(\Delta PQR\right) = ar\left(\Delta PSR\right)$

25. Show that ar$\left(\Delta ABG\right) = \frac{1}{3} ar\left(\Delta ABC\right)$, if median of Δ intersect at G.



Ans. AD is median

$ar\left(\Delta ABD\right) = ar\left(\Delta ACD\right)..........(i)$

GD is median

$ar\left(\Delta GBD\right) = ar\left(\Delta GCD\right).........(ii)$

Subtracting (ii) and (i)

$ar\left(\Delta ABD\right) - ar\left(\Delta GBD\right) = ar\left(\Delta ACD\right) - ar\left(\Delta GCD\right)$

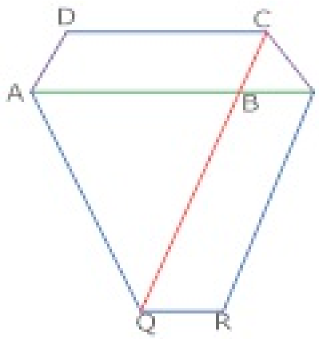
$ar\left(\Delta ABG\right) = ar\left(\Delta AGC\right)..........(iii)$

$ar\left(\Delta AGB\right) = ar\left(\Delta BGC\right)..........(iv)$

From (iii) and (iv)

$$ar(\Delta AGB) = \frac{1}{3}(ar\Delta ABC)$$

26. Show that ar (ABCD) = ar (BQRP), AQ is drawn Parallel to CP to intersect CB produced to Q and parallelogram BQRP is completed if P is any Point on AB produced.



Ans. AC is diagonal of ||gram ABCD

$$2ar(\Delta ABC) = ar(\|\ gramABCD)..........(i)$$

$$2ar(\Delta BPQ) = ar(\|\ gramBQRP).......(ii)$$

$$ar(\Delta AQC) = ar(\Delta AQP)$$
$$ar(\Delta AQC) - ar(\Delta BAQ) = (\Delta AQP) - ar(\Delta BAQ)$$
$$ar(\Delta ABC) = ar(\Delta BPQ).......(iii)$$

From (i) ,(ii) and (iii)

$$ar(\|\ gramABCD) = ar(\|\ gramBQRP)$$

27. Show that area of $\Delta BPQ = \frac{1}{2}$ area of ΔABC. D is mid-point of AB, P is any point on BC. PQ is joined and line CQ is drawn parallel to PD to intersect AB at Q.

Ans. CD is median

$ar(\Delta BCD) = \frac{1}{2} ar(\Delta ABC)$........$(i)$

$ar(\Delta PDQ) = ar(\Delta PDC)$..........$(ii)$

From (i)

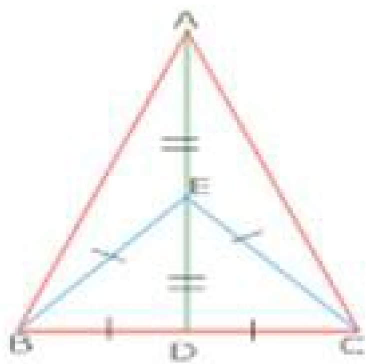
$ar(\Delta BCD) = \frac{1}{2} ar(\Delta ABC)$

$ar(\Delta BPD) + ar(\Delta PDC) = \frac{1}{2} ar(\Delta ABC)$

$ar(\Delta BPD) + ar(\Delta PDQ) = \frac{1}{2} ar(\Delta ABC)$..........$(ii)$

$ar(\Delta BPQ) = \frac{1}{2} ar(\Delta ABC)$

28. E is the mid-point of median AD, show that ar. $(BED) = \frac{1}{4} ar(ABC)$.



Ans. $ar(\Delta ABD) = ar(\Delta ACD)$

$ar(\Delta ABD) = \frac{1}{2} ar(\Delta ABC)$

Similarly, in $\Delta ABD, BE$ is the median

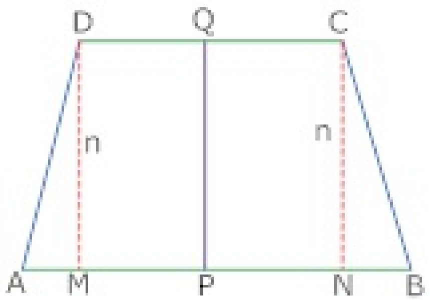
$ar(\Delta BED) = \frac{1}{2} ar(\Delta ABD)$

$$ar(\Delta BED) = \frac{1}{2} \times \frac{1}{2} ar(\Delta ABC)$$

$$= \frac{1}{4} ar(\Delta ABC)$$

29. Show that the line segments joining the mid-points of parallel sides of a trapezium divides it into two parts of equal area

Ans.



Draw $DM \perp AP$ and $CN \perp PB$

DM=CN=h

Area of trapezium APQD $= \frac{1}{2}(AP + DQ) \times DM$

$$= \frac{1}{2}\left[\frac{1}{2}AB + \frac{1}{2}CD\right] \times h$$

$$= \frac{1}{4}h(AB + CD) \text{........}(i)$$

Area of trapezium PBCQ

$$= \frac{1}{4}h(AB + CD)$$

From (i) and (2)

ar (trap. APQD)= ar (trap. PBCQ)

30. Prove that ar$(ADX) = ar(ACY)$. if AB||DC and line parallel to AC intersects AB at X and BC at Y

Ans. Join CX

$$\text{ar}(\Delta ACX) = \text{ar}(\Delta ACY)$$
$$\text{ar}(\Delta ACX) = \text{ar}(\Delta ADX)$$
$$\text{ar}(\Delta ACY) = \text{ar}(\Delta ADX)$$
$$\text{ar}(ADX) = \text{ar}(ACY)$$

31. Prove that area of ΔGBC = area of quadrilateral AFGE if BE and CF medians intersect at G.

Ans. In ΔABE , BE is the median

Area (ΔBCE) =area (ΔABE)

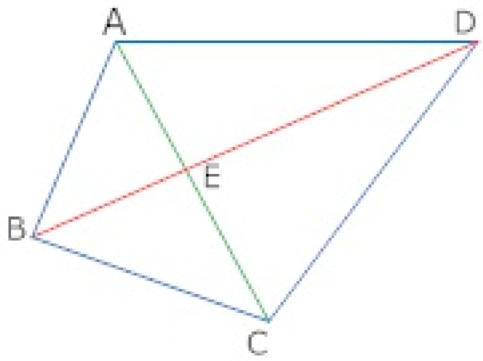
Area (ΔBGC) =area (ΔCGE) = area $(\text{quad.}AFGE)$ + area (ΔBGF)

Now, CF median of ΔABC ar (ΔBCF)=area (ΔACF)

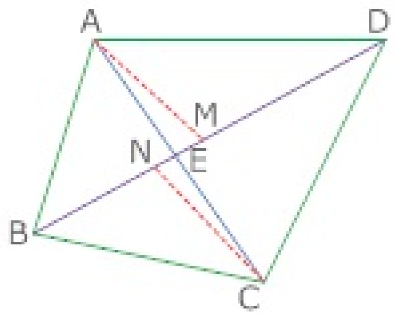
$$\text{area}(\Delta BGC) + \text{area}(\Delta BGF) = \text{area}(\text{quad.}AFGE) + \text{area}(AGGE)$$

$$2 \times \text{area}(\Delta BGC) = 2 \times \text{area}(\text{quad.}AFGE)$$
$$\text{area}(\Delta BGC) = \text{area}(\text{quad.}AFGE)$$

32. Show that ar $\Delta AED \times$ area $\Delta BEC =$ area $\Delta ABE \times$ area ΔCDE if diagonals of quadrilateral AC and BD intersect at a Point E.



Ans. Draw AM ⊥ BD and also CN ⊥ BD



$$ar(\Delta AED) \times ar(\Delta BEC) = \left(\frac{1}{2}ED \times AM\right) \times \left(\frac{1}{2}BE \times CN\right)$$

$$= \frac{1}{4}ED \times AM \times BE \times CN$$

$$= \left(\frac{1}{2}BE \times AM\right) \times \left(\frac{1}{2}ED \times CN\right)$$

$$= ar(\Delta ABE) \times ar(\Delta CDE)$$

$$ar(\Delta AED) \times ar(\Delta BEC) = ar(\Delta ABE) \times ar(\Delta CDE)$$

CBSE Class 9 Mathemaics

Important Questions

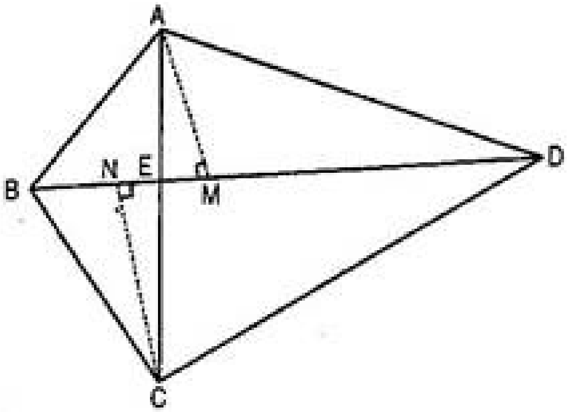
Chapter 9

Areas of Parallelograms and Triangles

4 Marks Quetions

1. Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that:

$ar\ (APB) \times ar\ (CPD) = ar\ (APD) \times ar\ (BPC)$



Ans. Given: A quadrilateral ABCD, in which diagonals

AC and BD intersect each other at point E.

To Prove: $ar\ (AED) \times ar\ (BEC)$

$= ar\ (ABE) \times ar\ (CDE)$

Construction: From A, draw AM $\perp$ BD and from C, draw CN $\perp$ BD.

Proof: ar (Δ ABE) = $\frac{1}{2} \times BE \times AM$ ....(i)

And ar (Δ AED) = $\frac{1}{2} \times DE \times AM$ ....(ii)

Dividing eq. (ii) by (i), we get,

$$\frac{ar(\Delta AED)}{ar(\Delta ABE)} = \frac{\frac{1}{2} \times DE \times AM}{\frac{1}{2} \times BE \times AM}$$

$$\Rightarrow \frac{ar(\Delta AED)}{ar(\Delta ABE)} = \frac{DE}{BE} \quad ....(iii)$$

Similarly $\frac{ar(\Delta CDE)}{ar(\Delta BEC)} = \frac{DE}{BE}$ ....(iv)

From eq. (iii) and (iv), we get

$$\frac{ar(\Delta AED)}{ar(\Delta ABE)} = \frac{ar(\Delta CDE)}{ar(\Delta BEC)}$$

$$\Rightarrow ar\ (AED) \times ar\ (BEC)$$

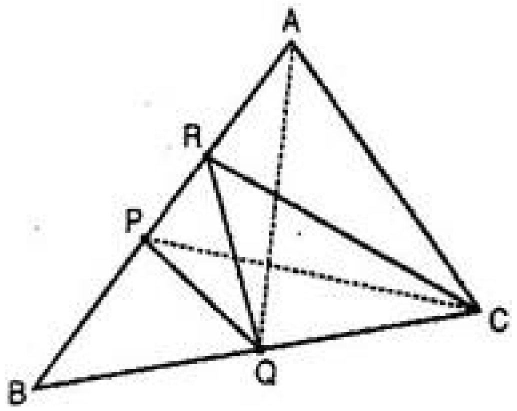
$$= ar\ (ABE) \times ar\ (CDE)$$

2. P and Q are respectively the mid-points of sides AB and BC or a triangle ABC and R is the mid-point of AP, show that:

(i) ar (PRQ) = $\frac{1}{2}$ ar (ARC)

(ii) ar (RQC) = $\frac{3}{8}$ ar (ABC)

(iii) ar (PBQ) = ar (ARC)



Ans. (i) PC is the median of ΔABC.

$\therefore$ ar (ΔBPC) = ar (ΔAPC) ....(i)

RC is the median of ΔAPC.

$\therefore$ ar (ΔARC) = $\frac{1}{2}$ ar (ΔAPC) ....(ii)

[Median divides the triangle into two triangles of equal area]

PQ is the median of ΔBPC.

$\therefore$ ar (ΔPQC) = $\frac{1}{2}$ ar (ΔBPC) ....(iii)

From eq. (i) and (iii), we get,

ar (ΔPQC) = $\frac{1}{2}$ ar (ΔAPC) ....(iv)

From eq. (ii) and (iv), we get,

ar (ΔPQC) = ar (ΔARC) ....(v)

We are given that P and Q are the mid-points of AB and BC respectively.

$\therefore$ PQ $\parallel$ AC and PA = $\frac{1}{2}$ AC

$\Rightarrow$ ar (ΔAPQ) = ar (ΔPQC) ....(vi) [triangles between same parallel are equal in area]

From eq. (v) and (vi), we get

ar (ΔAPQ) = ar (ΔARC) ....(vii)

R is the mid-point of AP. Therefore RQ is the median of ΔAPQ.

$\therefore$ ar (ΔPRQ) = $\frac{1}{2}$ ar (ΔAPQ) ....(viii)

From (vii) and (viii), we get,

ar (ΔPRQ) = $\frac{1}{2}$ ar (ΔARC)

(ii) PQ is the median of ΔBPC

$\therefore$ ar (ΔPQC) = $\frac{1}{2}$ ar (ΔBPC) = $\frac{1}{2}$ x $\frac{1}{2}$ ar (ΔABC) = $\frac{1}{4}$ ar (ΔABC) ....(ix)

Also ar (ΔPRC) = $\frac{1}{2}$ ar (ΔAPC) [Using (iv)]

$\Rightarrow$ ar (ΔPRC) = $\frac{1}{2}$ x $\frac{1}{2}$ ar (ΔABC) = $\frac{1}{4}$ ar (ΔABC) ....(x)

Adding eq. (ix) and (x), we get,

ar (ΔPQC) + ar (ΔPRC) = $\left(\frac{1}{4}+\frac{1}{4}\right)$ ar (ΔABC)

$\Rightarrow$ ar (quad. PQCR) = $\frac{1}{2}$ ar (ΔABC) ....(xi)

Subtracting ar (ΔPRQ) from the both sides, ar (quad. PQCR) – ar (Δ PRQ) = $\frac{1}{2}$ ar (Δ ABC) – ar (Δ PRQ)

$\Rightarrow$ ar (Δ RQC) = $\frac{1}{2}$ ar (Δ ABC) – $\frac{1}{2}$ ar (Δ ARC) [Using result (i)]

$\Rightarrow$ ar (Δ ARC) = $\frac{1}{2}$ ar (Δ ABC) – $\frac{1}{2} \times \frac{1}{2}$ ar (Δ APC)

$\Rightarrow$ ar (Δ RQC) = $\frac{1}{2}$ ar (Δ ABC) – $\frac{1}{4}$ ar (Δ APC)

$\Rightarrow$ ar (Δ RQC) = $\frac{1}{2}$ ar (Δ ABC) – $\frac{1}{4} \times \frac{1}{2}$ ar (Δ ABC) [PC is median of Δ ABC]

$\Rightarrow$ ar (Δ RQC) = $\frac{1}{2}$ ar (Δ ABC) – $\frac{1}{8}$ ar (Δ ABC)

$\Rightarrow$ ar (Δ RQC) = $\left(\frac{1}{2} - \frac{1}{8}\right)$ x ar (Δ ABC)

$\Rightarrow$ ar (Δ RQC) = $\frac{3}{8}$ ar (Δ ABC)

(iii) ar (Δ PRQ) = $\frac{1}{2}$ ar (Δ ARC) [Using result (i)]

$\Rightarrow$ 2 ar (Δ PRQ) = ar (Δ ARC) ..(xii)

ar (Δ PRQ) = $\frac{1}{2}$ ar (Δ APQ) [RQ is the median of Δ APQ] ....(xiii)

But ar (Δ APQ) = ar (Δ PQC) [Using reason of eq. (vi)] ....(xiv)

From eq. (xiii) and (xiv), we get,

ar (Δ PRQ) = $\frac{1}{2}$ ar (Δ PQC) ....(xv)

But ar (Δ BPQ) = ar (Δ PQC) [PQ is the median of Δ BPC] ....(xvi)

From eq. (xv) and (xvi), we get,

ar (ΔPRQ) = $\frac{1}{2}$ ar (ΔBPQ) ....(xvii)

Now from (xii) and (xvii), we get,

$2\left(\frac{1}{2}ar(\Delta BPQ)\right)$ = ar (ΔARC) $\Rightarrow$ ar (ΔBPQ) = ar (ΔARC)

3. In figure, ABC is a right triangle right angled at A. BCED, ACFG and ABMN are squares on the sides BC, CA and AB respectively. Line segment AX $\perp$ DE meets BC at Y. Show that:

(i) ΔMBC $\cong$ ΔABD

(ii) ar (BYXD) = 2 ar (MBC)

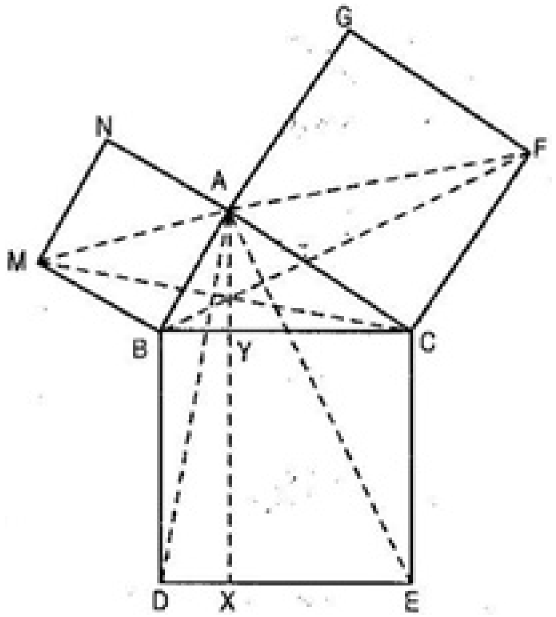
(iii) ar (BYXD) = ar (ABMN)

(iv) ΔFCB $\cong$ ΔACE

(v) ar (CYXE) = 2 ar (FCB)

(vi) ar (CYXE) = ar (ACFG)

(vii) ar (BCED) = ar (ABMN) + ar (ACFG)



Ans. (i) $\angle$ABM = $\angle$CBD = 90°

Adding $\angle$ABC both sides, we get,

$\angle$ABM + $\angle$ABC = $\angle$CBD + $\angle$ABC

$\Rightarrow \angle MBC = ABD$ ....(i)

Now in $\triangle MBC$ and $\triangle ABD$,

MB = AB [equal sides of square ABMN]

BC = BD [sides of square BCED]

$\angle MBC = \angle ABD$ [proved above]

$\therefore \triangle MBC \cong \triangle ABD$ [By SAS congruency]

(ii) From above, $\triangle MBC \cong \triangle ABD$

$\Rightarrow$ ar ($\triangle$MBC) = ar ($\triangle$ABD) $\Rightarrow$ ar ($\triangle$MBC) = ar (trap. ABDX) – ar ($\triangle$ADX)

$\Rightarrow$ ar ($\triangle$MBC) = $\frac{1}{2}$ (BD + AX) BY – $\frac{1}{2}$ DX.AX

$\Rightarrow$ ar ($\triangle$MBC) = $\frac{1}{2}$ BD.BY + $\frac{1}{2}$ AX.BY – $\frac{1}{2}$ DX.AX

$\Rightarrow$ ar ($\triangle$MBC) = $\frac{1}{2}$ BD.BY + $\frac{1}{2}$ AX (BY – DX)

$\Rightarrow$ ar ($\triangle$MBC) = $\frac{1}{2}$ BD.BY + $\frac{1}{2}$ AX. 0 [BY = DX]

$\Rightarrow$ ar ($\triangle$MBC) = $\frac{1}{2}$ BD.BY

$\Rightarrow$ 2 ar ($\triangle$MBC) = BD.BY $\Rightarrow$ 2 ar ($\triangle$MBC) = ar (rect. BYXD)

Hence ar (BYXD) = 2 ar ($\triangle$MBC)

(iii) Join AM. ABMN is a square.

Therefore, NA $\parallel$ MB $\Rightarrow$ AC $\parallel$ MB

Now $\triangle$AMB and $\triangle$MBC are on the same base and between the same parallels MB and AC.

$\therefore$ ar ($\triangle$AMB) = ar ($\triangle$MBC) ....(ii)

From result (ii), we have ar (BYXD) = 2 ar ($\triangle$MBC) ....(iii)

Using eq. (ii) and (iii), we get, ar (BYXD) = 2 ar ($\triangle$AMB)

$\Rightarrow$ ar (BYXD) = ar (square ABMN)

[Diagonal AM of square ABMN divides it in two triangles of equal area]

(iv) In $\triangle$FCB and $\triangle$ACE,

FC = AC [sides of square ACFG]

BC = CE [sides of square BCED]

$\angle BCF = \angle ACE$ [$\because \angle ACF = \angle BCE = 90^\circ$]

Adding $\angle$ ACB both sides,

$\angle$ BCF + $\angle$ ACB = $\angle$ ACE + $\angle$ ACB $\Rightarrow$ $\angle$ BCF = $\angle$ ACE

$\therefore$ ΔFCB $\cong$ ΔACE [By SAS congruency]

(v) From (iv), we have, ΔFCB $\cong$ ΔACE

$\Rightarrow$ ar (ΔFCB) = ar (ΔACE) $\Rightarrow$ ar (ΔFCB) = ar (trap. ACEX) – ar (ΔAEX)

$\Rightarrow$ ar (ΔFCB) = $\frac{1}{2}$ (CE + AX) CY – $\frac{1}{2}$ XE.AX

$\Rightarrow$ ar (ΔFCB) = $\frac{1}{2}$ CE.CY + $\frac{1}{2}$ AX.CY – $\frac{1}{2}$ XE.AX

$\Rightarrow$ ar (ΔFCB) = $\frac{1}{2}$ CE.CY + $\frac{1}{2}$ AX (CY – XE)

$\Rightarrow$ ar (ΔFCB) = $\frac{1}{2}$ CE.CY + $\frac{1}{2}$ AX. 0 [CY = XE]

$\Rightarrow$ ar (ΔFCB) = $\frac{1}{2}$ CE.CY

$\Rightarrow$ 2 ar (ΔFCB) = CE.CY $\Rightarrow$ 2 ar (ΔFCB) = ar (rect. CYXE)

Hence ar (BYXD) = 2 ar (ΔFCB)

(vi) Join AF. ACFG is a square.

$\therefore$ FC $\parallel$ AG $\Rightarrow$ FC $\parallel$ AB

Now ΔACF and ΔFCB are on the same base FC and between the same parallels FC and AB.

$\therefore$ ar (ΔACF) = ar (ΔFCB) ....(v)

From result (v), we get, ar (CYXE) = 2 ar (ΔFCB) ....(vi)

Using eq. (v) in (vi), we get, ar (CYXE) = 2 ar (ΔACF)

Diagonal AF of square ACFG divides it in two triangles of equal area.

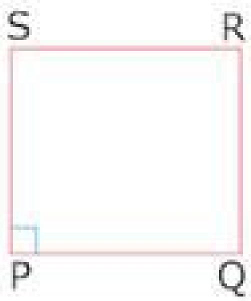
$\therefore$ ar (CYXE) = ar (sq. ACFG) ....(vii)

(vii) Adding eq. (iv) and (vii), we get,

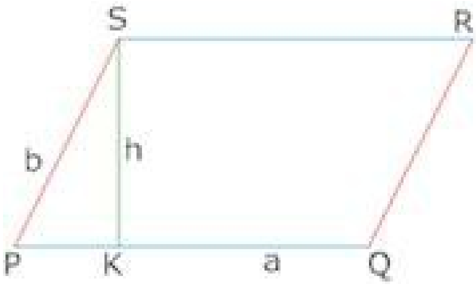
ar (BYXD) + ar (CYXE) = ar (ABMN) + ar (ACFG)

$\Rightarrow$ ar (BCED) = ar (ABMN) + ar (ACFG)

4. Prove that the parallelogram which is a rectangle has the greatest area.



Ans. Let PQRS be a parallelogram in which PQ = a and PS = b and h be the altitude corresponding to base PQ



Area of parallelogram PQRS = Base $\times$ corresponding Altitude = ah

ΔPSK is a right angled triangle $b(PS)$ being its hypotenuse.

But hypotenuse is the greatest side of Δ

Area of (ah) of ||gram PQRS will be greatest when h is greatest

H = b, then PS $\perp$ PQ

The ||gram PQRS will be a rectangle.

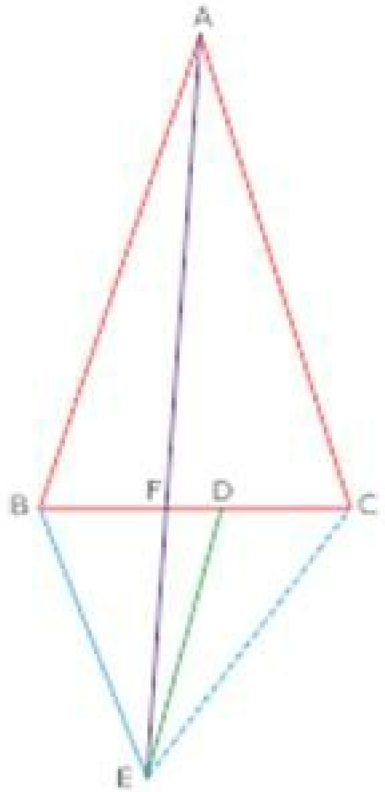
Hence, the area of ||gram is greatest when it is a rectangle.

5. Prove that

(i) ar(ΔBDE)= $\frac{1}{4}$ ar(ΔABC)

(ii) ar(ΔBDE)= $\frac{1}{2}$ ar(ΔBAE)

If ΔABC and ΔDBE are two equilateral triangles such that D is the mid-point of BC and AE intersects BC at F.



Ans. Join EC

(i) let a be the side of equilateral ΔABC

$$ar(\Delta ABC)=\frac{\sqrt{3}}{4}a^2\,........(i)$$

$$ar(\Delta BDE)=\frac{\sqrt{3}}{4}\left(\frac{a}{2}\right)^2$$

$$=\frac{\sqrt{3}}{16}a^2\,........(ii)$$

From (i) and (ii)

$$ar(\Delta BDE)=\frac{1}{4}ar(\Delta ABC)$$

(ii) $ar(\Delta BDE)=\frac{1}{2}ar(\Delta BEC)$

$$\angle EBC=60^\circ$$

$\angle BCA = 60°$

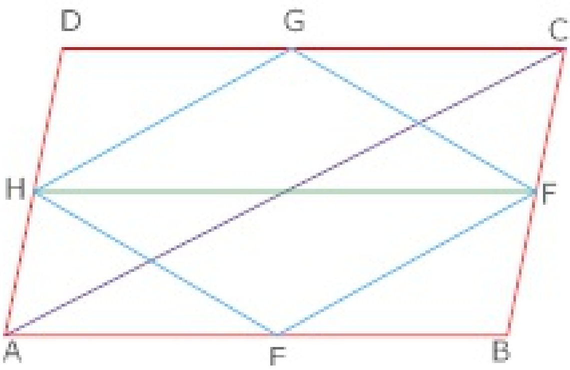
$\angle EBC = \angle BCA$

$BE \| AC$

$ar(\Delta BEC) = ar(\Delta BAE)$

$ar(BDE) = \frac{1}{2} ar(\Delta BAE)$

6. Show that EFGH is a ||gram and its area is half of the area of ||gram ABCD. If E, F, G, H are respectively the mid points of the sides AB, BC, CD and DA.



Ans. Join AC and HF

E and F are the mid-points of AB and BC

$\therefore$ EF= $\frac{1}{2}$ AC and EF||AC..........(i)

Similarly, GH = $\frac{1}{2}$ AC and GH||AC.........(ii)

From (i) and (ii)

GH = EF and GH||EF

$\therefore$ EFGH is a ||gram

ar $(\Delta HGF) = \frac{1}{2} ar(\| \, gram \; HDFC)$ ...(iii)

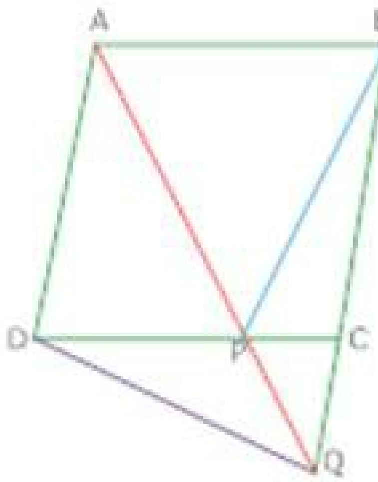
$\text{ar}\left(\Delta HEF\right)=\frac{1}{2}ar\left(\|\ gram\ HABF\right)$ ......(iv)

Adding (iii) and (iv),

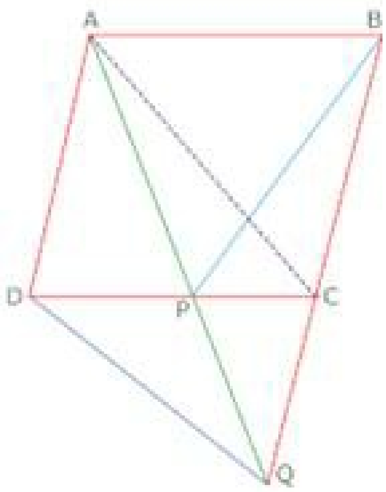
$\text{ar}\ \left(\Delta HGF\right)+ar\left(\Delta HEF\right)=\frac{1}{2}ar\left(\|\ gram\ HDCF\right)+ar\left(\|\ gram\ HABF\right)$

$\Rightarrow ar\left(\|\ gram\ EFGH\right)\frac{1}{2}\text{are}\left(\|\ gram\ ABCD\right)$

7. Show that ar (BPC) = ar (DPQ) if BC is produced to a point Q such that AD = CQ and AQ intersect DC at P



Ans. Join AC



$ar\left(\Delta BCP\right)=ar\left(\Delta APC\right)...(i)$

AD=CQ

$AD \| BC$

$AD \| CQ$

Hence, a pair of opposite side AD and CQ of the quadrilateral ADQC is equal and parallel.

In ΔAPC and ΔQPD,

AP=QP

CP=DP

$\angle APC = \angle QPD$

$\Delta APC \cong \Delta QPD$

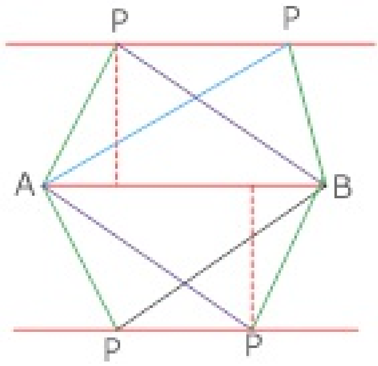
$ar(\Delta APC) = ar(\Delta QPD)..(ii)$

From (i) and (ii)

$ar(\Delta BCP) = ar(\Delta QPD)$

$ar(BPC) = ar(DPQ)$

8. If area of $\Delta PAB = K$ and two points A and B are positive real number K. find the lows of a point p



Ans. Let the perpendicular distance of P from AB be h

$ar(\Delta PAB) = K$

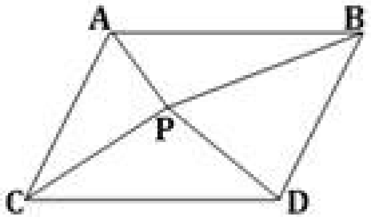
$\frac{1}{2} \times (AB) \times h = K$

$h = \frac{2K}{AB}$

Since AB and K are given h is a fixed Positive real number. This means that P lies on a line Parallel to AB at a distance h from it.

Hence, the locus of P is a pair of lines at a distance $h = \frac{2K}{AB}$, parallel to AB.

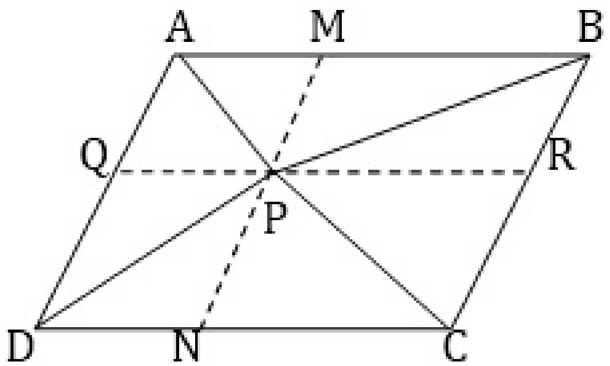
9. In figure, P is a point in the interior of a parallelogram ABCD. Show that:



(i) ar (APB) + ar (PCD) = $\frac{1}{2}$ ar (ABCD)

(ii) ar (APD) + ar (PBC) = ar (APB) + ar (PCD)

Ans. (i) Draw a line passing through point P and parallel to AB which intersects AD at Q and BC at R respectively.



Now $\triangle$APB and parallelogram ABRQ are on the same base AB and between same parallels AB and QR.

$\therefore$ ar ($\triangle$APB) = $\frac{1}{2}$ ar ($\|$gm ABRQ) ....(i)

Also $\triangle$PCD and parallelogram DCRQ are on the same base AB and between same parallels AB and QR.

$\therefore$ ar ($\triangle$PCD) = $\frac{1}{2}$ ar ($\|$gm DCRQ) ....(ii)

Adding eq. (i) and (ii),

ar ($\triangle$APB) + ar ($\triangle$PCD) = $\frac{1}{2}$ ar ($\|$gm ABRQ) + $\frac{1}{2}$ ar ($\|$gm DCRQ)

$\Rightarrow$ ar ($\triangle$APB) = $\frac{1}{2}$ ar ($\|$gm ABCD) ....(iii)

(ii) Draw a line through P and parallel to AD which intersects AB at M and DC at N.

Now $\triangle$APD and parallelogram AMND are on the same base AD and between same parallels AD and MN.

$\therefore$ ar ($\triangle$APD) = $\frac{1}{2}$ ar (|| gm AMND) ....(iv)

Also $\triangle$PBC and parallelogram MNCB are on the same base BC and between same parallels BC and MN.

$\therefore$ ar ($\triangle$PBC) = $\frac{1}{2}$ ar (|| gm MNCB) ....(v)

Adding eq. (i) and (ii),

ar ($\triangle$APD) + ar ($\triangle$PBC) = $\frac{1}{2}$ ar (|| gm AMND) + $\frac{1}{2}$ ar (|| gm MNCB)

$\Rightarrow$ ar ($\triangle$APD) = $\frac{1}{2}$ ar (|| gm ABCD) ....(vi)

From eq. (iii) and (vi), we get,

ar ($\triangle$APB) + ar ($\triangle$PCD) = ar ($\triangle$APD) + ar ($\triangle$PBC)

or ar ($\triangle$APD) + ar ($\triangle$PBC) = ar ($\triangle$APB) + ar ($\triangle$PCD)

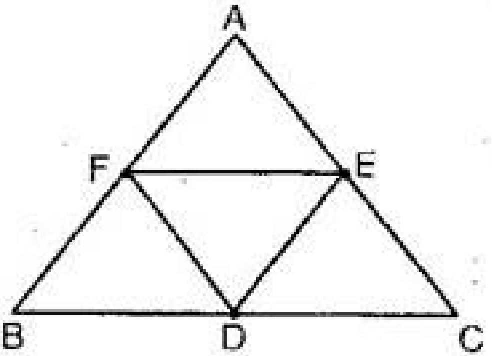
Hence proved.

10. D, E and F are respectively the mid-points of the sides BC, CA and AB of a $\triangle$ABC. Show that:

(i) BDEF is a parallelogram.

(ii) ar (DEF) = $\frac{1}{4}$ ar (ABC)

(iii) ar (BDEF) = $\frac{1}{2}$ ar (ABC)



Ans. (i) F is the mid-point of AB and E is the mid-point of AC.

$\therefore$ FE || BC and FE = $\frac{1}{2}$ BD

[$\because$ Line joining the mid-points of two sides of a triangle is parallel to the third and half of it]

$\Rightarrow$ FE || BD [BD is the part of BC]

And FE = BD

Also, D is the mid-point of BC.

$\therefore$ BD = $\frac{1}{2}$ BC

And FE || BC and FE = BD

Again E is the mid-point of AC and D is the mid-point of BC.

$\therefore$ DE || AB and DE = $\frac{1}{2}$ AB

$\Rightarrow$ DE || AB [BF is the part of AB]

And DE = BF

Again F is the mid-point of AB.

$\therefore$ BF = $\frac{1}{2}$ AB

But DE = $\frac{1}{2}$ AB

$\therefore$ DE = BF

Now we have FE $\parallel$ BD and DE $\parallel$ BF

And FE = BD and DE = BF

Therefore, BDEF is a parallelogram.

(ii) BDEF is a parallelogram.

$\therefore$ ar ($\triangle$ BDF) = ar ($\triangle$ DEF) ....(i) [diagonals of parallelogram divides it in two triangles of equal area]

DCEF is also parallelogram.

$\therefore$ ar ($\triangle$ DEF) = ar ($\triangle$ DEC) ....(ii)

Also, AEDF is also parallelogram.

$\therefore$ ar ($\triangle$ AFE) = ar ($\triangle$ DEF) ....(iii)

From eq. (i), (ii) and (iii),

ar ($\triangle$ DEF) = ar ($\triangle$ BDF) = ar ($\triangle$ DEC) = ar ($\triangle$ AFE) ....(iv)

Now, ar ($\triangle$ ABC) = ar ($\triangle$ DEF) + ar ($\triangle$ BDF) + ar ($\triangle$ DEC) + ar ($\triangle$ AFE) ....(v)

$\Rightarrow$ ar ($\triangle$ ABC) = ar ($\triangle$ DEF) + ar ($\triangle$ DEF) + ar ($\triangle$ DEF) + ar ($\triangle$ DEF) [Using (iv) & (v)]

$\Rightarrow$ ar ($\triangle$ ABC) = 4 x ar ($\triangle$ DEF)

$\Rightarrow$ ar ($\triangle$ DEF) = $\frac{1}{4}$ ar ($\triangle$ ABC)

(iii) ar ($\parallel$ gm BDEF) = ar ($\triangle$ BDF) + ar ($\triangle$ DEF) = ar ($\triangle$ DEF) + ar ($\triangle$ DEF) [Using (iv)]

$\Rightarrow$ ar ($\|$gm BDEF) = 2 ar ($\triangle$ DEF)

$\Rightarrow$ ar ($\|$gm BDEF) = 2 x $\frac{1}{4}$ ar ($\triangle$ ABC)

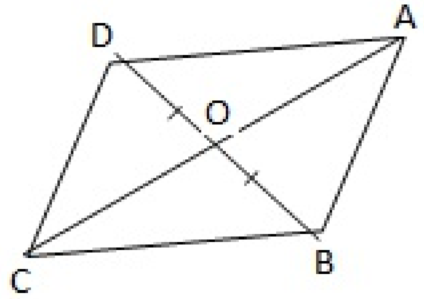
$\Rightarrow$ ar ($\|$gm BDEF) = $\frac{1}{2}$ ar ($\triangle$ ABC)

11. In figure, diagonals AC and BD of quadrilateral ABCD intersect at O such that OB = OD. If AB = CD, then show that:

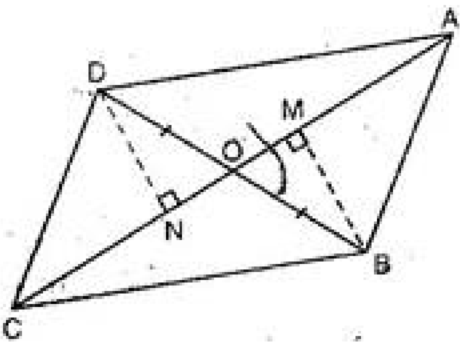
(i) ar (DOC) = ar (AOB)

(ii) ar (DCB) = ar (ACB)

(iii) DA $\|$ CB or ABCD is a parallelogram.



Ans. (i) Draw BM $\perp$ AC and DN $\perp$ AC.



In $\triangle$ DON and $\triangle$ BOM,

OD = OB [Given]

$\angle$ DNO = $\angle$ BMO = 90° [By construction]

$\angle$ DON = $\angle$ BOM [Vertically opposite]

$\therefore$ $\triangle$ DON $\cong$ $\triangle$ BOM [By RHS congruency]

$\Rightarrow$ DN = BM [By CPCT]

Also ar ($\triangle$ DON) = ar ($\triangle$ BOM) ....(i)

Again, In $\triangle$ DCN and $\triangle$ ABM,

CD = AB [Given]

$\angle$ DNC = $\angle$ BMA = 90° [By construction]

DN = BM [Prove above]

$\therefore$ $\triangle$ DCN $\cong$ $\triangle$ BAM [By RHS congruency]

$\therefore$ ar ($\triangle$ DCN) = ar ($\triangle$ BAM) ....(ii)

Adding eq. (i) and (ii),

ar ($\triangle$ DON) + ar ($\triangle$ DCN) = ar ($\triangle$ BOM) + ar ($\triangle$ BAM)

$\Rightarrow$ ar ($\triangle$ DOC) = ar ($\triangle$ AOB)

(ii) Since ar ($\triangle$ DOC) = ar ($\triangle$ AOB)

Adding ar $\triangle$ BOC both sides,

ar ($\triangle$ DOC) + ar $\triangle$ BOC = ar ($\triangle$ AOB) + ar $\triangle$ BOC

$\Rightarrow$ ar ($\triangle$ DCB) = ar ($\triangle$ ACB)

(iii) Since ar ($\triangle$ DCB) = ar ($\triangle$ ACB)

Therefore, these two triangles in addition to be on the same base CB lie between two same parallels CB and DA.

$\therefore$ DA $\parallel$ CB

Now AB = CD and DA $\parallel$ CB

Therefore, ABCD is a parallelogram.

12. In figure, ABC and BDF are two equilateral triangles such that D is the mid-point of BC. If AE intersects BC at F, show that:

(i) ar (BDE) = $\frac{1}{4}$ ar (ABC)

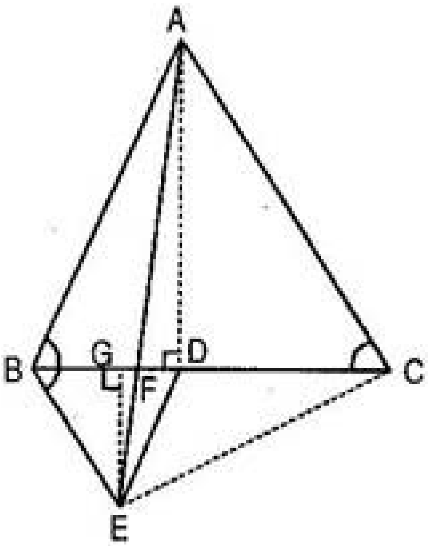
(ii) ar (BDE) = $\frac{1}{2}$ ar (BAE)

(iii) ar (ABC) = 2 ar (BEC)

(iv) ar (BFE) = ar (AFD)

(v) ar (BFE) = 2 ar (FED)

(vi) ar (FED) = $\frac{1}{8}$ ar (AFC)



Ans. Join EC and AD.

Since $\triangle$ABC is an equilateral triangle.

$\therefore \angle A = \angle B = \angle C = 60^\circ$

Also $\triangle$BDE is an equilateral triangle.

$\therefore \angle B = \angle D = \angle E = 60^\circ$

If we take two lines, AC and BE and BC as a transversal.

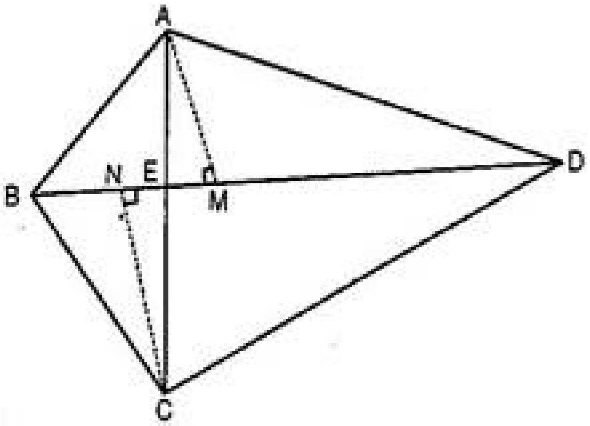
Then $\angle B = \angle C = 60^\circ$ [Alternate angles]

$\Rightarrow$ BE $\parallel$ AC

Similarly, for lines AB and DE and BF as transversal.

Then $\angle B = \angle C = 60^\circ$ [Alternate angles]

$\Rightarrow$ BE $\parallel$ AC



(i) Area of equilateral triangle BDE = $\frac{\sqrt{3}}{4}(BD)^2$ ....(i)

Area of equilateral triangle ABC = $\frac{\sqrt{3}}{4}(BC)^2$ ....(ii)

Dividing eq. (i) by (ii),

$$\frac{ar(\Delta BDE)}{ar(\Delta ABC)} = \frac{\frac{\sqrt{3}}{4}(BD)^2}{\frac{\sqrt{3}}{4}(BC)^2}$$

$$\Rightarrow \frac{ar(\Delta BDE)}{ar(\Delta ABC)} = \frac{\frac{\sqrt{3}}{4}(BD)^2}{\frac{\sqrt{3}}{4}(2BD)^2} \quad [\because BD = DC]$$

$$\Rightarrow \frac{ar(\Delta BDE)}{ar(\Delta ABC)} = \frac{(BD)^2}{(2BD)^2}$$

$$\Rightarrow \frac{ar(\Delta BDE)}{ar(\Delta ABC)} = \frac{1}{4}$$

$\Rightarrow$ ar (Δ BDE) = $\frac{1}{4}$ ar (Δ ABC)

(ii) In Δ BEC, ED is the median.

$\therefore$ ar (Δ BEC) = ar (Δ BAE) ....(i)

[Median divides the triangle in two triangles having equal area]

Now BE $\parallel$ AC

And Δ BEC and Δ BAE are on the same base BE and between the same parallels BE and AC.

$\therefore$ ar (Δ BEC) = ar (Δ BAE) ....(ii)

Using eq. (i) and (ii), we get

Ar (Δ BDE) = $\frac{1}{2}$ ar (Δ BAE)

(iii) We have ar (Δ BDE) = $\frac{1}{4}$ ar (Δ ABC) [Proved in part (i)] ....(iii)

ar (Δ BDE) = $\frac{1}{4}$ ar (Δ BAE) [Proved in part (ii)]

ar (Δ BDE) = $\frac{1}{4}$ ar (Δ BEC) [Using eq. (iii)] ....(iv)

From eq. (iii) and (iv), we het

$\frac{1}{4}$ ar (Δ ABC) = $\frac{1}{4}$ ar (Δ BEC)

$\Rightarrow$ ar (Δ ABC) = 2 ar (Δ BEC)

(iv) Δ BDE and Δ AED are on the same base DE and between same parallels AB and DE.

$\therefore$ ar (Δ BDE) = ar (Δ AED)

Subtracting Δ FED from both the sides,

ar (Δ BDE) – ar (Δ FED) = ar (Δ AED) – ar (Δ FED)

$\Rightarrow$ ar (Δ BFE) = ar (Δ AFD) ....(v)

(v) An in equilateral triangle, median drawn is also perpendicular to the side,

$\therefore$ AD $\perp$ BC

Now ar (Δ AFD) = $\frac{1}{2} \times FD \times AD$ ....(vi)

Draw EG $\perp$ BC

$\therefore$ ar (Δ FED) = $\frac{1}{2} \times FD \times EG$ ....(vii)

Dividing eq. (vi) by (vii), we get

$$\frac{\text{ar}(\Delta AFD)}{\text{ar}(\Delta FED)} = \frac{\frac{1}{2} \times FD \times AD}{\frac{1}{2} \times FD \times EG}$$

$$\Rightarrow \frac{\text{ar}(\Delta AFD)}{\text{ar}(\Delta FED)} = \frac{AD}{EG}$$

$$\Rightarrow \frac{\text{ar}(\Delta AFD)}{\text{ar}(\Delta FED)} = \frac{\frac{\sqrt{3}}{4} BC}{\frac{\sqrt{3}}{4} BD}$$ [Altitude of equilateral triangle = $\frac{\sqrt{3}}{4}$ side]

$$\Rightarrow \frac{ar(\Delta AFD)}{ar(\Delta FED)} = \frac{2BD}{BD}$$ [D is the mid-point of BC]

$$\Rightarrow \frac{ar(\Delta AFD)}{ar(\Delta FED)} = 2$$

$\Rightarrow$ ar (Δ AFD) = 2 ar (Δ FED) ......(viii)

Using the value of eq. (viii) in eq. (v),

Ar (Δ BFE) = 2 ar (Δ FED)

(vi) ar (Δ AFC) = ar (Δ AFD) + ar (Δ ADC) = 2 ar (Δ FED) + $\frac{1}{2}$ ar (Δ ABC) [using (v)

= 2 ar (Δ FED) + $\frac{1}{2}$ [4 x ar (Δ BDE)] [Using result of part (i)]

= 2 ar (Δ FED) + 2 ar (Δ BDE) = 2 ar (Δ FED) + 2 ar (Δ AED)

[Δ BDE and Δ AED are on the same base and between same parallels]

= 2 ar (Δ FED) + 2 [ar (Δ AFD) + ar (Δ FED)]

= 2 ar (Δ FED) + 2 ar (Δ AFD) + 2 ar (Δ FED) [Using (viii)]

= 4 ar (Δ FED) + 4 ar (Δ FED)

$\Rightarrow$ ar (Δ AFC) = 8 ar (Δ FED)

$\Rightarrow$ ar (Δ FED) = $\frac{1}{8}$ ar (Δ AFC)